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Non-Instantaneous Adiabats in Finite Time

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1. Introduction

Since the pioneer paper of Curzon and Ahlborn (1975), the so called finite time thermodynamics has been in development. Curzon and Ahlborn proposed a model of thermal engine called endoreversible cycle or Curzon and Ahlborn cycle, shown in Figure 1, with the so called Curzon-Ahlborn-Novikov efficiency,

$$\eta_{CAN} = 1 - \sqrt{T_C / T_H}\ , \tag{1}$$

where T_C is the cold reservoir temperature and T_H is the hot reservoir temperature. This endoreversible cycle is an engine in which the endoreversibility condition, $Q_H / T_{HW} = Q_C / T_{CW}$, is fulfilled and the entropy production during the exchange of heat, Q_H and Q_C, between the system and its reservoirs of heat is only taken into account. The temperatures of working substance are T_{HW} and T_{CW}. The relation between these temperatures in the cycle is $T_C < T_{CW} < T_{HW} < T_H$.

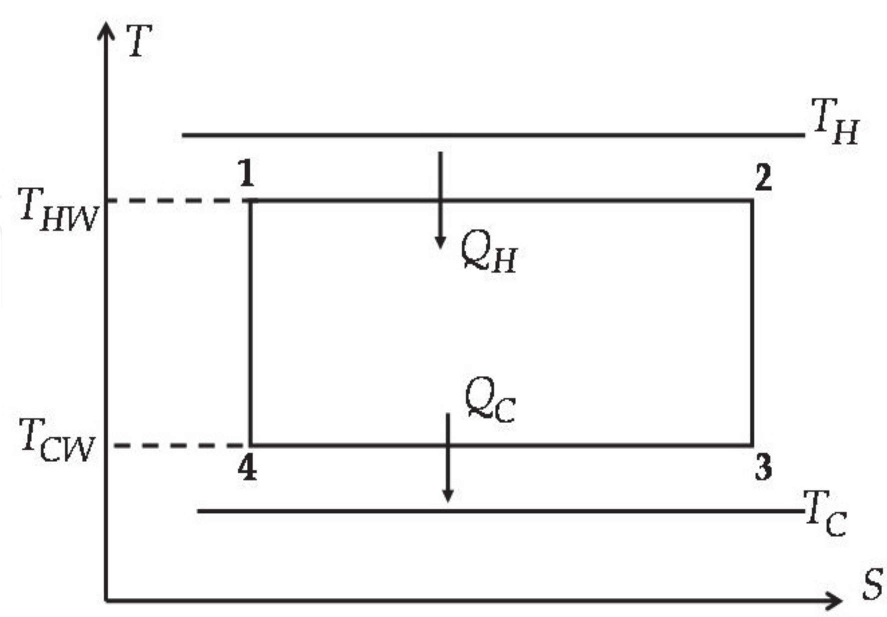

Fig. 1. Curzon and Ahlborn cycle in the entropy *S* vs temperature *T* plane.

As we can see, Carnot efficiency, η_C , is obtained when the temperatures of reservoirs are the same as engine temperatures, which means $T_{HW} = T_H$ and $T_C = T_{CW}$ in Figure 1, namely,

$$\eta_C = 1 - \frac{T_C}{T_H} = 1 - \frac{T_{CW}}{T_{CH}} \tag{2}$$

Equation (1) was previously advanced by Chambadal (1957) and Novikov (1958), among others, and has been recovered by some procedures (Salamon, et al., 1976; Rubin, 1979a, 1979b, 1980; Bejan, 1996; Gutkowicz-Krusin et al., 1978 among others). Particularly, the optimal configuration of heat engines was studied (Rubin, 1979a), and it was introduced a procedure in which the power output of cycle is taken as a function of the compression ratio by using the parameter $\lambda \sim \left[\ln V_{\max} - \ln V_{\min}\right]^{-1}$, where $V_{\max}$ and $V_{\min}$ are the maximum and the minimum volumes spanned in the cycle, respectively (Gutkowicz-Krusin et al., 1978). More recently, this subject has been also studied by other authors (Badescu, 2004; Amelkin, et al., 2004, 2005; Song et al., 2006, 2007). Even more Angulo-Brown (1991) introduced an optimization criterion of merit for the Curzon and Ahlborn cycle taking into account entropy production, the ecological criterion, through the function,

$$E = P - T_C \sigma \, , \tag{3}$$

where P is the power output, T_C is the temperature of cold reservoir an σ is the total entropy production. The function in (3) is known as ecological function, and at maximum of this function the efficiency of Curzon and Ahlborn cycle can be written as,

$$\eta_E = 1 - \sqrt{(\varepsilon^2 + \varepsilon)/2} \, . \tag{4}$$

A comparison of values obtained with the previous expressions of efficiency, for some plants reported in the literature of finite time thermodynamics, is shown in Table 1.

Notice that the ecological criterion proposed by Angulo-Brown for finite-time Carnot heat engines, Equation (3), represents a compromise between the high power output P and a loss power output, $T_C \sigma$. However Yan (1993) showed that it might be more reasonable to use $E_0 = P - T_0 \sigma$ if the cold reservoir temperature T_C is not equal to the environments temperature T_0 because in the definition of E two different quantities, exergy output, P, and a non-exergy $T_C \sigma$, were compared together. The criterion with function E_0 is more reasonable than that presented by Angulo-Brown. Nevertheless, since $E_0 \to E$ when $T_0 \to T_C$ it can be used as the optimization of E without loss of generality.

Recently, following the procedure of Gutkowicz-Krusin et al (1978) the form of the ecological function and its efficiency was found using the Newton heat transfer law and ideal gas as working substance (Ladino-Luna & de la Selva, 2000), and using Dulong-Petit heat transfer law for ideal gas as working substance (Ladino-Luna,2003).

It is important to remark that Curzon and Ahlborn efficiency is an adequate approximation for conventional power plants, and ecological efficiency is the adequate approximation for modern power plants (nuclear and others), as it is shown in Table 1.

On other hand, in nature there are no endoreversible engines. Thus, some authors have analyzed the non-endoreversible Curzon and Ahlborn cycle. Ibrahim et al. (1991), and Wu and Kiang (1992) proposal include a non-endoreversibility parameter to take into account

Plant	T_C / T_H	η_C	η_{CAN}	η_E	η_{obs}
West Thurrock (coal fired steam plant), U K	298/838	0.64439	0.40367	0.50905	0.360
Lardarello (geothermal steam plant), Italy	353/523	0.32505	0.17845	0.24818	0.160
Central steam power station, U K	298/698	0.57307	0.3466	0.44809	0.280
Steam power plant, U S A	298/923	0.67714	0.43179	0.55447	0.400
Combined-cycle (steam and mercury), U S A	298/783	0.61941	0.38308	0.48744	0.340
Doel 4 (nuclear pressurized water reactor), Bélgium	283/566	0.50000	0.29289	0.38763	0.350
Almaraz II (nuclear pressurized water reactor), Spain	290/600	0.51667	0.30478	0.40127	0.345
Sizewell B (nuclear pressurized water reactor), U K	288/581	0.50430	0.29594	0.39114	0.363
Cofrentes (nuclear boiling water reactor), Spain	289/562	0.48577	0.28290	0.37603	0.340
Heysham (nuclear advanced gas cooled reactor), U K	288/727	0.60385	0.37060	0.47413	0.400

Table 1. Values of different efficiency expressions for the cycle in Figure 1. T is in Kelvin scale.

internal irreversibilities in the cycle. Later, Chen (1994, 1996) analyzed the effect of thermal resistances, heat leakage and internal irreversibility with these parameter definite as,

$$I_S \equiv \Delta S_C \,/\, \Delta S_H \,, \tag{5}$$

where ΔS_C is the entropy change during heat exchange from the engine to the cold reservoir, and ΔS_H is the entropy change during heat exchange from the hot reservoir to the engine. Chen et. al. (2004, 2006) carried out the ecological optimization for generalized irreversible Carnot engine with heat resistance, heat leakage and internal irreversibility for newtonian heat transfer law. Zhu et. al (2003) used a generalized convective heat transfer law $Q \propto (\Delta T)^n$, and generalized radiative heat transfer law $Q \propto \Delta(T^n)$. More recently the ecological optimization for generalized irreversible universal heat engine, including Diesel, Otto, Bryton Atkinson, Dual and Miller cycles, with heat resistance, heat leakage and internal irreversibility was carried out for newtonian heat transfer law (Chen et al., 2007). The non-endoreversible Curzon and Ahlborn cycle model is shown in Figure 2. The efficiency of Curzon and Ahlborn cycle using the parameter in Equation (5), at maximum power output was found as (Chen, 1994, 1996),

$$\eta_m = 1 - \sqrt{I_S \varepsilon}\,, \quad I_S > 1 \,. \tag{6}$$

On other hand, Angulo-Brown et al (1999) showed that a general property of endoreversible Curzon and Ahlborn cycle demostrated previously (Árias-Hernández & Angulo-Brown, 1997) can be extended for a non-endoreversible Curzon and Ahlborn cycle. Besides, Velasco et. al. (2000) follow the idea in Chen (1994, 1996), and they found expressions to measure possible reductions of non-desired effects in heat engines operation. They pointed out that I_S is not depending of ε and re-wrote Equation (6) as,

$$\eta_m = 1 - \sqrt{\varepsilon / I}, \quad I \equiv 1/I_S, \quad 0 < I < 1. \tag{7}$$

Even more, Angulo-Brown et. al. (2002) applied variational calculus to show that both the saving function (Velasco et. al., 2000) and a modified ecological criterion are equivalent.
These previous results have been found assuming an ideal gas as working fluid. However, in a real context, a thermal engine works with a non-ideal gas. The performannce of a finite time cycle with a van der Waals gas as working fluid was analyzed among others by Agrawal & Menon (1990), and more recently by Ladino-Luna (2005).

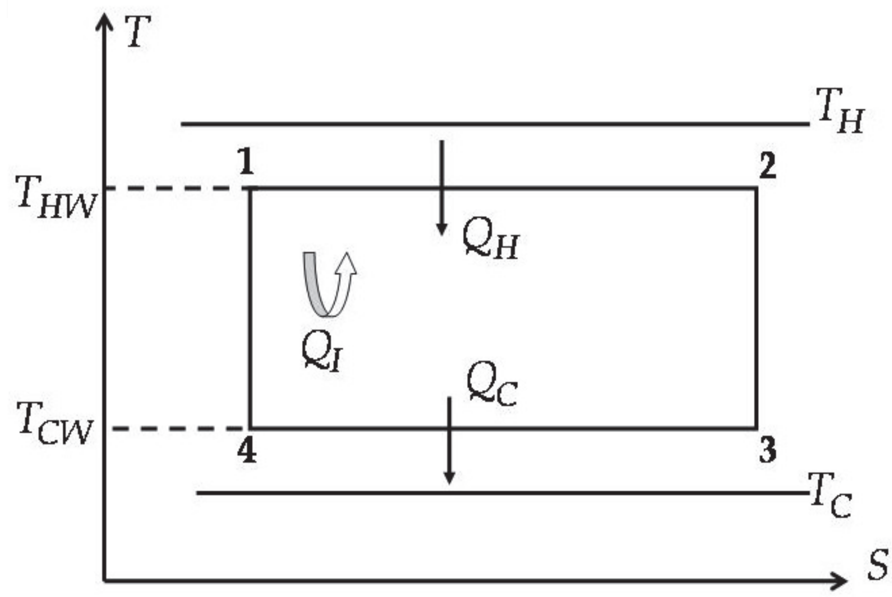

Fig. 2. Non-endoreversible Curzon and Ahlborn cycle in the *S*-*T* plane. Q_I is the generated heat by internal processes.

In the present work it is shown that some of internal irreversibilities can be taken into account for a more general expressions of both power output and ecological function, with a non-linear heat transfer law like $\frac{dQ}{dt} \propto (\Delta T)^k$, assuming the Curzon and Ahlborn cycle with non-instantaneous adiabats. Approximate efficiencies curves at maximum power output and at maximum ecological function are shown for $k = \frac{5}{4}$, that is in case of the Dulong and Petit heat transfer law. Comparative tables of values of efficiencies are shown for certain power plants reported in some papers, (Árias-Hernández & Angulo-Brown, 1997; Velasco et al., 2000 and more recently Ladino-Luna, 2008). The cycle analysis shall be treated assuming both ideal gas and van der Waals gas as working fluids. Also, we show that power output and ecological function have a similar property when the compression ratio is taking into account, e. g. the efficiency obtained at maximum of each of objective function with the definition $\varepsilon = T_C / T_H$ is a bound for efficiencies, when an engine is modeled as a Curzon and Ahlborn cycle working at maximum of each objetive function and the time of all the processes in the cycle is took into account (Ladino-Luna & de la Selva, 2000; Ladino Luna, 2003). All of quantities have been taken in the International Measurement System.

2. Non-instananeous adiabats with Newton´s heat transfer law

From classical thermodynamics, the efficiency of a reversible thermal engine working between two temperatures $T_{HW} > T_{CW}$ is known when heat exchanged is also known. In this description, the temperatures of working gas in the isothermal processes, T_{HW} and T_{CW}, are assumed to be the same as that of the corresponding reservoirs. As a consequence the process associated with the heat transfer between the engine and the reservoirs is ignored. The upper limit of the efficiency of any heat engine corresponds to the Carnot cycle, η_C, in which the temperatures of the reservoirs are the same as the temperatures of the heat engine in Figure 1, as it was shown in Equation (2). Thus, the definition of efficiency of an engine working in cycles leads to the Carnot efficiency, fulfilling,

$$\frac{Q_H - Q_C}{Q_H} \leq 1 - \frac{T_{CW}}{T_{HW}} \tag{8}$$

It is important to note that the following expressions, with the change $\varepsilon \equiv T_C / T_H$, Carnot efficiency $\eta_C \equiv 1-\varepsilon$, Curzon and Ahlborn-Novikov efficiency $\eta_{CAN} = 1-\sqrt{\varepsilon}$ and ecological efficiency $\eta_E = 1-\sqrt{(\varepsilon^2+\varepsilon)/2}$, and using the Newton heat transfer law, appear as a function of ε, so they can be written in general as,

$$\eta = 1 - z(\varepsilon)\,, \tag{9}$$

Thus, the problem of finding the efficiency of a heat engine modeled as a Curzon and Ahlborn cycle, and using either of two alternatives, maximizing power output or maximizing ecological function, becomes the problem of finding a function $z = z(\varepsilon)$, as complicated or simple as to allow the heat transfer law being used and the chosen procedure. So, substituting $z = z(\varepsilon)$ in expression (9) the efficiency is obtained as

$$\eta = \eta(\varepsilon)\,. \tag{10}$$

So we can use the Newton's heat transfe law, and also we can assume the same thermal conductance α in two isothermal processes of Curzon and Ahlborn cycle. The heat exchanged between the engine and its surroundings can be expressed as,

$$\frac{dQ_H}{dt} = \alpha(T_H - T_{HW}) \qquad \text{and} \qquad \frac{dQ_H}{dt} = \alpha(T_C - T_{CW})\,. \tag{11}$$

2.1 The Gutkowics-Krusin, Procaccia and Ross model

With the previous ideas, to make the present paper self-contained we include in this section a brief explanation and some results of model used by Gutkowicz-Krusin et al (1978), and others that we need for our present purposes. In their model Gutkowicz-Kru et al. consider a working substance inside of a cylinder with a movable piston as engine, and also they considered an ideal gas as working fluid, contained in the cylinder and the mass of piston not negligible. The inertia of the movable piston does not affect the endoreversible character of Curzon and Ahlborn cycle to consider the expansion of gas, and because the volume occupied by the gas in the expansion and compression can be written as

$$V = l \cdot A\,, \tag{12}$$

where V is the volume occupied by the gas, A is the cross section area (constant) of the cylinder and l is the distance traveled by the piston in the expansion or compression of gas. The acceleration of the piston during the processes is

$$\frac{d^2 l}{dt^2} = \frac{1}{A} \cdot \frac{d^2 V}{dt^2}\,, \tag{13}$$

so that, from the pressure definition,

$$pressure = \frac{force}{area}\,, \tag{14}$$

and with the Newton's second law, namely $force = (mass) \cdot (acceleration)$, we can write,

$$\frac{1}{A}\left(m \frac{d^2 l}{dt^2} \right) = \frac{force}{area} = pressure\,. \tag{15}$$

For the *gas+piston* system, the difference in internal and external pressures is expressed by

$$p - p_{ext} = \frac{m}{A} \frac{d^2 V}{dt^2}\,. \tag{16}$$

On other hand, conservation of energy law of the system can be written as

$$\frac{dU}{dt} = \frac{dQ}{dt} - p_{ext} \frac{dV}{dt} - \frac{m}{A} \frac{d^2 V}{dt^2} \frac{dV}{dt}\,, \tag{17}$$

where the last term represents the power output during the movement of piston to take volumen V. Substituting (16) in (17) it is obtain

$$\frac{dU}{dt} = \frac{dQ}{dt} - p_{ext} \frac{dV}{dt} - (p - p_{ext}) \frac{dV}{dt} \quad ó \quad \frac{dU}{dt} = \frac{dQ}{dt} - p \frac{dV}{dt}\,, \tag{18}$$

which means that the system is in mechanical equilibrium with its surroundings. Now, we can find the form of time for each process using the Newton's heat transfer law, Equation (11). For isothermal processes, using an ideal gas we have $U = U(T) = constant$, indicating that (18) is reduced to the expression,

$$\frac{dQ}{dt} = p \frac{dV}{dt}\,. \tag{19}$$

Due to the equation of state for ideal gas, (19) can be written as

$$\frac{dQ}{dt} = \frac{RT}{V} \frac{dV}{dt} = RT \frac{d}{dt} (\ln V)\,; \tag{20}$$

The power, defined by the quotient of the total work output W and the total time t_{tot} is as,

$$P = \frac{W}{t_{tot}} = \frac{\alpha(T_{HW} - T_{CW})\left(\ln\frac{V_3}{V_1} + \frac{1}{\gamma-1}\ln\frac{T_{CW}}{T_{HW}}\right)}{\left(\frac{T_{HW}}{T_H - T_{HW}} + \frac{T_{CW}}{T_{CW} - T_C}\right)\ln\frac{V_3}{V_1}}. \tag{21}$$

α is the thermal conductance, $\gamma \equiv C_P / C_V$; t_{tot} is the cycle period and the adiabatic processes are not instantaneous. In fact,

$$t_{TOT} = t_1 + t_2 + t_3 + t_4 \tag{22}$$

where the times for the isothermal processes have been found to be,

$$t_1 = \frac{RT_{HW}}{\alpha(T_H - T_{HW})}\ln\frac{V_2}{V_1} \text{ and } t_3 = \frac{RT_{CW}}{\alpha(T_{CW} - T_C)}\ln\frac{V_4}{V_3} \tag{23}$$

and the times for the adiabatic processes have been assumed to be:

$$t_2 = f_1 \ln\frac{V_3}{V_2} \text{ and } t_4 = f_2 \ln\frac{V_4}{V_1}, \tag{24}$$

with

$$f_1 \equiv \frac{RT_{HW}}{\alpha(T_H - T_{HW})} \text{ and } f_2 \equiv \frac{RT_{CW}}{\alpha(T_{CWw} - T_C)}, \tag{25}$$

where R is the general constant of gases. The heat flows, Q_H and Q_C are assumed to be given by Newton's heat transfer law, as (11). The power output is written in terms of the variables $u = T_{HW} / T_H$ and $z = T_{CW} / T_{HW}$ from which we obtain $P = P(u,z)$ as,

$$P = \frac{\alpha T_H (1-z)\left[1 + \lambda \ln z\right]}{\frac{1}{1-u} + \frac{z}{uz-\varepsilon}}, \tag{26}$$

and its maximization conditions $\partial P / \partial u = 0$ and $\partial P / \partial z = 0$ allow to obtain

$$u = \frac{z+\varepsilon}{2z}, \tag{27}$$

and

$$(z^2 - \varepsilon)(1 + \lambda \ln z) = \lambda(z-\varepsilon)(1-z); \tag{28}$$

where λ represents the external parameter,

$$\lambda = \frac{1}{(\gamma-1)\ln(V_3 / V_1)} \tag{29}$$

meaning that

$$P_{\max} = P_{\max}(u(z), z), \tag{30}$$

that is $P_{\max}$ is a projection on the (z, P) plane. It is also found that at the maximum power condition z is given by a power series in λ:

$$z_P = \sqrt{\varepsilon} + \tfrac{1}{2}(1-\sqrt{\varepsilon})^2\lambda + \tfrac{1}{4}(1-\sqrt{\varepsilon})^2\left[(1-\sqrt{\varepsilon})^2 / 2\sqrt{\varepsilon} - \ln\varepsilon\right]\lambda^2 + O(\lambda^3) \tag{31}$$

Upon substiting Equation (31) in Equation (9) and because the terms in the series (31) are positive, an upper bound for the efficiency is obtained when $\lambda = 0$, i.e. when the engine size goes towards infinity, it is the following one:

$$\eta_{\max} = 1 - z_P(\lambda = 0) = \eta_{CAN} \tag{32}$$

In the next section we construct the equation analogous to (31) for the ecological function following the Gutkowicz-Krusin, Procaccia and Ross model outlined here.

2.2 The ecological function

In the ecological function, Equation (3), we take P from Equation (26) and the entropy production term σ as $\sigma = \Delta S / t_{tot}$, where ΔS represents the entropy change caused at the isothermal processes because of the heat transfers Equation (11),

$$\sigma = \frac{1}{t_{tot}}\left(\frac{Q_C}{T_C} - \frac{Q_H}{T_H}\right). \tag{33}$$

t_{tot} is given by Equations (22) to (25), and in terms of the variables (u, z, ε), σ becomes,

$$\sigma = \alpha\frac{T_1}{T_2}\frac{\left(\ln\frac{V_3}{V_1} + \frac{1}{\gamma-1}\ln z\right)(z-\varepsilon)}{\ln\frac{V_3}{V_1}\left[\frac{1}{1-u} + \frac{z}{zu-\varepsilon}\right]}, \tag{34}$$

where, thanks to the endoreversibility condition, we have used the thermostatic results $V_2 / V_1 = V_3 / V_4$ and $V_2 = V_3\left(T_{CW} / T_{HW}\right)^{\frac{1}{\gamma-1}}$, where λ is given by Equation (29).
With Equations(26) and (34) the expression for the ecological function becomes

$$E = \alpha T_1\frac{(1+\varepsilon-2z)(1+\lambda\ln z)}{\frac{1}{1-u} + \frac{z}{zu-\varepsilon}} \tag{35}$$

Figure 3 shows the behavior of $P / \alpha T_H$, $\sigma / \alpha T_H$ and $E / \alpha T_H$ in the u constant plane, at $\lambda = 0$ and ε a given constant value. It is apparent that the maximum power output is achieved with high production of entropy, it is also apparent that zero entropy production is achieved with zero power output, while the function E represents the maximum possible power output with the minimum possible entropy production.
Upon maximizing the two variables function $E = E(u, z)$ (ε defined positive and λ defined semipositive, being external parameters), we obtain for $\partial E / \partial u = 0$ and $\partial E / \partial z = 0$, at first $u = u(z)$, as in case of maximizing power output, and later the following relation between the variables z and u,

$$\left[2(1+\lambda\ln z)z - \lambda(1+\varepsilon-2z)\right](z-\varepsilon)(zu-\varepsilon) = (1+\varepsilon-2z)(1+\lambda\ln z)(1-u)\varepsilon z\,. \tag{36}$$

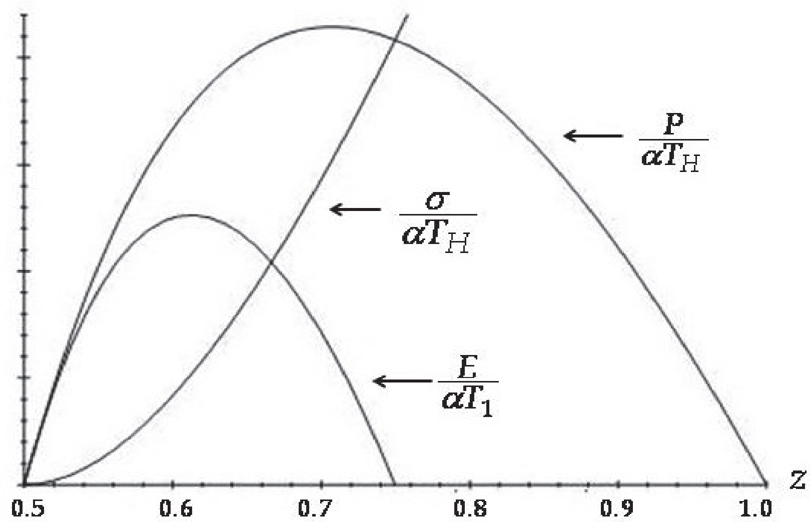

Fig. 3. Behavior of power output, entropy production and ecological efficiency. Here, $T_H = 400K, \varepsilon = 0.75, \lambda = 0$.

Substituting u from (27) in Equation (36) it is obtain the equation that z obeys at the maximum of the ecological function, namely,

$$\left[2(1+\lambda \ln z)z - \lambda(1+\varepsilon-2z)\right](z-\varepsilon) = (1+\varepsilon-2z)(1+\lambda \ln z)\varepsilon \,. \tag{37}$$

If we suppose $z \equiv z(\lambda)$ given by the power expansion,

$$z_P \equiv b_0 + b_1\lambda + b_2\lambda^2 + b_3\lambda^3 + \dots , \tag{38}$$

we find, upon taking the implicit successive derivatives of z_P with respect to λ in Equation (37) and equating them with the coefficients b_i in Equation (38),

$$\begin{aligned} z_E = \sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}\Bigg\{1 + \left[\tfrac{1}{4}(1+3\varepsilon)\sqrt{\tfrac{2}{\varepsilon+\varepsilon^2}} - 1\right]\lambda + \left(\frac{\frac{1}{16}(1+3\varepsilon)}{\varepsilon+\varepsilon^2} - \frac{1}{2}\sqrt{\frac{2}{\varepsilon+\varepsilon^2}}\ln\sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}\right)\times \\ \left(1+3\varepsilon - 4\sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)}\right)\lambda^2 + O(\lambda^3)\Bigg\} \end{aligned} \tag{39}$$

Furthermore, using (39), we can write the efficiency as a power series in λ ,

$$\eta_E \equiv 1 - z_E(\varepsilon, \lambda) \tag{40}$$

In the particular case when $\lambda = 0$ we find the value

$$z_{EO}(\varepsilon, \lambda = 0) = \sqrt{\frac{(\varepsilon+\varepsilon^2)}{2}} \,, \tag{41}$$

and the corresponding value for the ecological efficiency with instantaneous adiabats is as:

$$\eta_{EO} = 1 - z_{EO}(\varepsilon, \lambda = 0) = 1 - \sqrt{\tfrac{1}{2}(\varepsilon+\varepsilon^2)} \,, \tag{42}$$

which is the maximum possible one, since all the terms in Eq.(39) are positive.

2.3 The linear approximation

As we can see in Equations (31) and (39), it can be taken a linear approximation for the efficiency η in terms of compression ratio, namely $V_{\max} / V_{\min}$, and of the ratio T_C / T_H, obtaining an expression like $F(\eta, V_{\max}/V_{\min}, T_C/T_H) = 0$, with the same form regardless it was obtained by maximization of power output or maximization of ecological function. It permits analyze the behavior of compression ratio in respect to T_C / T_H. It can verified that $r_C \to \infty$ and $\lambda \to 0$ lead to the Curzon-Ahlborn-Novikov efficiency, now written as $\eta_{CAN} \equiv \eta_P(\lambda = 0) = \eta_{PO}$. From (31) the linear approximation can be obtained,

$$\eta_{PL}(\lambda) = 1 - \sqrt{\varepsilon} - \tfrac{1}{2}(1 - \sqrt{\varepsilon})^2 \lambda , \tag{43}$$

and the corresponding linear approximation of ecological efficiency is as,

$$\eta_{EL}(\lambda) = 1 - \sqrt{\tfrac{1}{2}(\varepsilon^2 + \varepsilon)} - \left[\tfrac{1}{4}(1 + 3\varepsilon) - \sqrt{\tfrac{1}{2}(\varepsilon^2 + \varepsilon)} \right] \lambda . \tag{44}$$

As can be seen, the linear approximation of efficiency, maximizing power output or ecological function, has the form,

$$\eta_{JL}(\varepsilon, \lambda) = \eta_{JO} - b_J(\varepsilon)\lambda = \eta_{JO} - \frac{b_J(\varepsilon)}{(\gamma - 1)\ln r_C}, \tag{45}$$

where b_J is de coefficient of linear term in λ, being $\lambda = \left[(\gamma - 1)\ln r_C\right]^{-1}$, and the subscript J is substituting by *P* or *E*, for each of cases: maximization of power output or maximization of ecological function. That is, for maximum power output we have η_{PL}, η_P and b_P; and for maximum ecological function we have η_{EL}, η_E and b_E. So, for a particular value of efficiency we have $r_C = r_C(\varepsilon)$. The general expression of $r_C(\varepsilon)$, from (45), is obtain as,

$$r_C = \exp\left\{ \frac{b_J}{(\gamma - 1)(\eta_{JO} - \eta_{JL})} \right\} . \tag{46}$$

Taking η_{JO} as Curzon and Ahlborn-Novikov efficiency or ecological efficiency, it is true,

$$0 < \eta_{JL} < \eta_{JO} . \tag{47}$$

A particular value of efficiency η_{JL} permits find the interval $0 < \varepsilon < 1$ in which r_C satisfies,

$$r_C > 1 , \tag{48}$$

and Equation (48) permits find, from (46),

$$\frac{b_J}{(\gamma - 1)(\eta_{JO} - \eta_{JL})} > 0 , \tag{49}$$

which leads to inequality

$$` \eta_{JL} < \eta_{IO} , \tag{50}$$

as a necessary condition because η_{JO} must be an upper bound for η_{JL}. For monoatomic gases $\gamma = 1.67$, with η_{JL} as a variable parameter, the variation of r_C can be obtained, and we can see that $r_C \to \infty$ when $\eta_{JL} \to \eta_{JO}$, as it should be. For each temperatures in Table 1 the variation of r_C is obtained from (46). By example in the West Turrock plant, $T_H = 838\,K$ and $T_C = 298\,K$, with $\eta_{PO} = \eta_{CA} = 0.403367$. Figure 4 shows the behavior of r_C respect to η_{PL}. Using the ecological function for the same plant, $\eta_{EO} = 0.50905$, and Figure 5 shows the behavior of r_C respect to η_{EL}. There is a minimum value of compression ratio greater than 1. On other hand for a particular value of r_C and for values of the used parameters in Figures 4 and 5, the behavior of η_{JL} can be considered as function of $\varepsilon \equiv T_C/T_H$, where we can be see the correctness of (50), so $\eta_{JL} \to \eta_{JO}$ only when $\varepsilon \to 1$, as it is shown in Figure 6, for the ecological function with $r_C = 10$, closer to compression values found in thermodynamics textbooks (among others Burghardt, 1982). In addition, the values of efficiency obtained naturally with the linear approximation are closer to real values than the corresponding values of η_{CA}, and η_E. The physically possible values of r_C take places when the values of λ that comply $0 < \lambda < 1$.

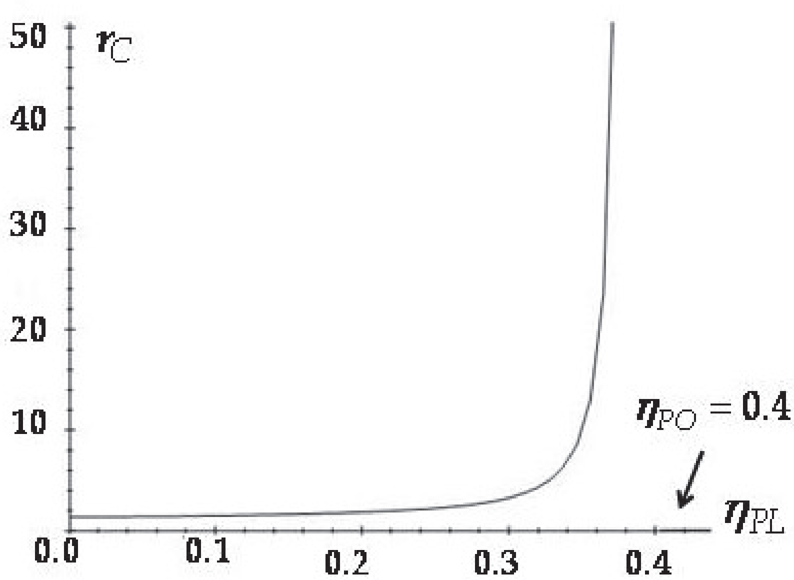

Fig. 4. Behavior of r_C in respect to variation of η_{PL} in the interval $[0, 0.403367)$.

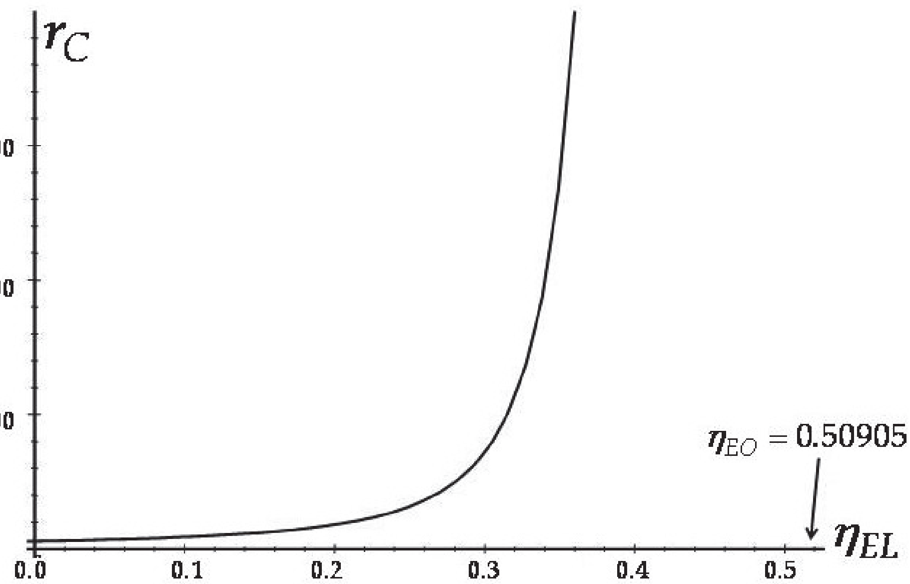

Fig. 5. Behavior of r_C in respect to variation of η_{EL} in the interval $[0, 0.50905)$.

In real compressors, named alternative compressors with dead space, percent of volume in the total displacement of a piston into a cylinder is named dead space ratio, defined as $c = (volume\ of\ dead\ space)/(volume\ of\ displacement)$, (Burghardt, 1982). In case of a Curzon and Ahlborn cycle $c = (\min imum\ volume)/(\max imum\ volume)$ is the reciprocal of r_C. Experimentally it is found that $3\% \le c \le 10\%$, so $100/3 \ge r_C \ge 100/10$, or $33 > r_C \ge 10$. Compression ratio is a useful parameter to model the behavior of a thermal engines, but it is not easy to include this parameter in design of power plants, would be interesting find a model in which r_C could be explicitly incorporated in design power plants.

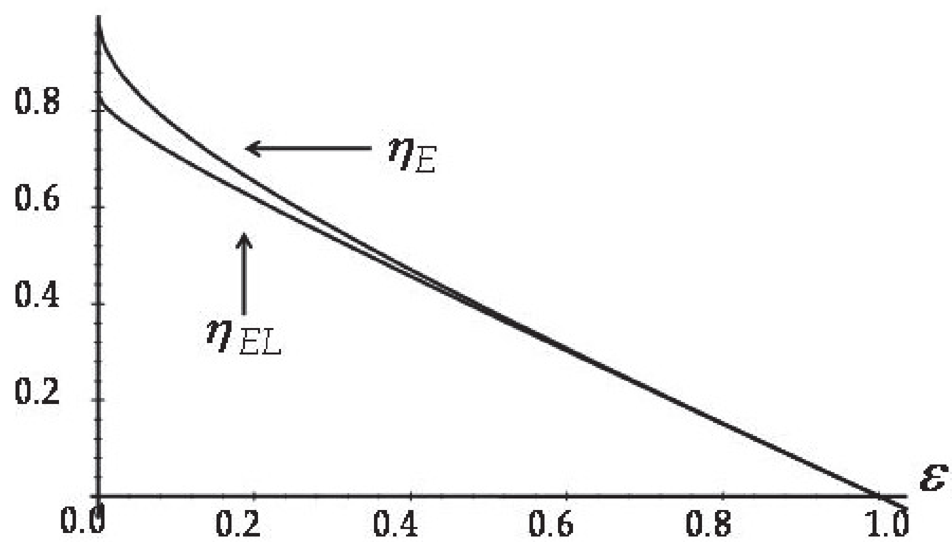

Fig. 6. Comparison between ecological efficiency at zero order and at linear order.

Supposing these plants are working as a Curzon and Ahlborn cycle, we found that linear approximation of efficiency, Equation (45), permits us to find intervals of efficiency values near to experimental values of efficiency than others. Table 2 shows a comparison between real values and linear approximation values, assuming ideal gas as working fluid ($\gamma = 1.67$), making clear the need for a closer approximation, nevertheless table shows the closeness of the linear approximation.

Nuclear power plant	$T_C\ (K)$	$T_H\ (K)$	η_{obs}	η_{EL} , $10 \le r_C < 33$
Doel 4 (Belgium),	283	566	0.35000	0.37944 to 0.38224
Almaraz II (Spain)	290	600	0.34500	0.39234 to 0.39539
Sizewell B, (U K)	288	581	0.36300	0.38277 to 0.38563
Cofrentes (Spain)	289	562	0.34000	0.36844 to 0.37103
Heysham (U K)	288	727	0.40000	0.46036 to 0.46506

Table 2. Comparison of values of experimental efficiencies and values of linear ecological approximation.

3. Non-instantaneous adiabats with Dulong-Petit's heat transfer law

The ecological efficiency has also been calculated using Dulong and Petit's heat transfer law (Angulo-Brown & Páez-Hernández, 1993; Arias-Hernández & Angulo-Brown, 1994), maximizing ecological function. Their numerical results have shown that the efficiency value changes with the heat transfer law one assumes. Velasco et al. (2000) studied both the power and the ecological function optimizations, using Newton's heat transfer law. It is worthwhile to point out that in all of the above quoted calculations the time for the adiabatic processes is not taken into account explicitly.

In the present section the power output P, and ecological function E are chosen to be maximized. Use is made of the more general Dulong and Petit heat transfer law, and the time for all the processes of the Curzon and Ahlborn cycle is explicitly taken into account, to see if the construction of a function $\eta_{PDP} = \eta_{PDP}(\lambda,\varepsilon)$ and $\eta_{EDP} = \eta_{EDP}(\lambda,\varepsilon)$ is possible. That the time for the adiabats can in principle to be an arbitrarily chosen function of the time of the isothems, here however it is chosen in the same way as in previous section for the purposes of comparison. The Dulong and Petit law has been chosen because the main ocurring heat transfers are conduction through the wall separating the working fluid from the thermal bath, and convection takes place within the working fluid. Radiative heat transfer is of smaller magnitude (O'Sullivan, 1990). With the optimization of the power output of a Curzon and Ahlborn engine, it is shown an approximate expression for efficiency by means of also the Dulong and Petit's heat transfer law, and the corresponding zero order term in a power series of the parameter λ above cited. We follow the procedure employed in the previous section.

3.1 The power output efficiency

Let us assume a gas in a cylinder with a piston as a working fluid that exchanges heat with the reservoirs like in previous section, and let us use a heat transfer law of the form:

$$\frac{dQ}{dt} = \alpha(T_f - T_i)^k \tag{51}$$

where $k > 1$, α is the thermal conductance which is assumed the same for both reservoirs, dQ/dt is the rate of heat Q exchange and T_i and T_f are the temperatures for the heat exchange process considered. From the first law of thermodynamics applied to gas under mechanical equilibrium condition, i.e., $p = p_{ext}$, we obtain

$$\frac{dU}{dt} = \frac{dQ}{dt} - p\frac{dV}{dt} , \tag{52}$$

and assuming an ideal gas as working substance $U = U(T)$. One has in case of isothermal processes $\frac{dU}{dt} = \frac{dU}{dV}\frac{dV}{dt} = 0$. Using Eq. (51) we obtain, for the isothermal processes that

$$\frac{dQ}{dt} = p\frac{dV}{dt} \quad \text{or} \quad \alpha(T_f - T_i)^k = \frac{RT_i}{V}\frac{dV}{dt} . \tag{53}$$

Equation (53) implies that the time of the process along the first isothermal process is

$$t_1 = \frac{RT_{HW}}{\alpha(T_H - T_{HW})^k}\ln\frac{V_2}{V_1} \tag{54}$$

and analogously, the time along the second isothermal process is

$$t_3 = \frac{RT_{CW}}{\alpha(T_{CW} - T_C)^k}\ln\frac{V_3}{V_4} , \tag{55}$$

The corresponding heat exchanged Q_H and Q_C become, respectively,

$$Q_H = RT_{HW}\ln\frac{V_2}{V_1} , \quad Q_C = RT_{CW}\ln\frac{V_4}{V_3} , \tag{56}$$

where, R is the general gas constant and $V_1, V_2, V_3, V_4,$ are the corresponding volumes for the states 1,2,3,4 in Figure 1.

While it is true that the speed for the adiabatic branches is independent from the speed of the isothermal ones in the cycle, but with a non null value, in order to obtain a more realistic result it will be assumed that their speed follows a similar law to the isothermal ones.

The previous assumption means that the rate of change of volume in the first adiabat is the same that in the first isotherm. Under this assumption, the time along the adiabatic processes is respectively,

$$t_2 = \frac{RT_{HW}}{\alpha(T_H - T_{HW})^k(\gamma-1)} \ln\frac{T_{HW}}{T_{CW}}, \text{ and } t_4 = \frac{-RT_{CW}}{\alpha(T_{CW}-T_C)^k(\gamma-1)} \ln\frac{T_{CW}}{T_{HW}}. \tag{57}$$

where $\gamma \equiv C_P / C_V$ has been used. With these results we can now compute the form for the power output, given by

$$P = \frac{W}{t_{tot}} = \frac{Q_1 + Q_2}{t_{tot}}, \tag{58}$$

where $t_{tot} = t_1 + t_2 + t_3 + t_4$. Power output is written as

$$t_{TOT} = \frac{R}{\alpha}\left[\frac{T_{HW}}{(T_H - T_{HW})^k} + \frac{T_{CW}}{(T_{CW} - T_C)^k}\right] \ln\frac{V_3}{V_1}, \tag{59}$$

by using $V_2 / V_1 = V_3 / V_4$ and $V_2 = V_3\left(T_{CW} / T_{HW}\right)^{\frac{1}{\gamma-1}}$; so that after making the exchange of variables as in Equation (26), P becomes,

$$P = \frac{T_1^k \alpha(1-z)(1+\lambda\ln z)}{\frac{1}{(1-u)^k} + \frac{z}{(zu-\varepsilon)^k}}, \tag{60}$$

with the same parameters as in previous section. By means of $\partial P / \partial u = 0$ and $\partial P / \partial z = 0$ we obtain,

$$u = \frac{z^{\frac{2}{k+1}} + \varepsilon}{z + z^{\frac{2}{k+1}}} \tag{61}$$

and,

$$\left[-z(1+\lambda\ln z)(zu-\varepsilon) + \lambda(1-z)(zu-\varepsilon) + zku(1-z)(1+\lambda\ln z)\right]\cdot\left((zu-\varepsilon)^k + z(1-u)^k\right)$$

$$-z(1-z)(1+\lambda\ln z)(zu-\varepsilon)\cdot\left(uk(zu-\varepsilon)^{k-1} + (1-u)^k\right) = 0 \tag{62}$$

Substituting the variable u in Equation (62) with the help of Eq. (61), the resulting expression is the following one, which shows the implicit function $z = z(\lambda, \varepsilon)$, for a given k,

$$\left[z^{\frac{2}{k+1}}(z-\varepsilon)\left(\lambda(1-z) - z(1+\lambda\ln z)\right) + zk(z^{\frac{2}{k+1}} + \varepsilon)(1-z)(1+\lambda\ln z)\right](z^{\frac{2k}{k+1}} + z)$$

$$-z(1-z)(1+\lambda\ln z)\left[z^2 + \varepsilon z^{\frac{2k}{k+1}} + z^{\frac{2}{k+1}}(z-\varepsilon)\right] = 0\,. \tag{63}$$

Because the solution of Eq. (63) is not analytically feasible when k is not an integer, the case discussed here is $k = 5/4$, the Dulong and Petit's heat transfer law. So one can take the reasonable approximations only for the exponents in Equation (63),

$$\frac{2}{k+1} \approx 1 \text{ and } \frac{2k}{k+1} \approx 1 \tag{64}$$

obtaining

$$(1+\lambda)((k\varepsilon + zk)(1-z) - z(z-\varepsilon)) + \lambda(1-z)(1-\varepsilon) - (1+\lambda \ln z)(1-z)z = 0 . \quad (65)$$

Equation (65) allows to obtain the explicit expression for the function $z = z(\varepsilon, k)$ for $\lambda = 0$,

$$z_{OP}(\varepsilon, k) = \frac{(k-1)(1-\varepsilon) \pm \sqrt{(\varepsilon-1)^2(1-k)^2 + 4k^2\varepsilon}}{2k} . \quad (66)$$

Taking now $k = 5/4$ in Equation (66) we obtain the following value for the physically acceptable solution of (63), namely,

$$z_{OPDP} = \frac{1-\varepsilon+\sqrt{\varepsilon^2+98\varepsilon+1}}{10} . \quad (67)$$

The numerical results for $\eta_{OPDP} = 1 - z_{OPDP}$ are shown in Table 3, compared with η_{CAN} and the observed efficiency η_{obs}, where can be seen that are in good agreement with the reported values. Figure 7 shows the comparison between η_{OPDP} and η_{CAN} with the temperatures of the reservoirs in real plants (Angulo-Brown & Páez-Hernández, 1993; Velasco et al., 2000).

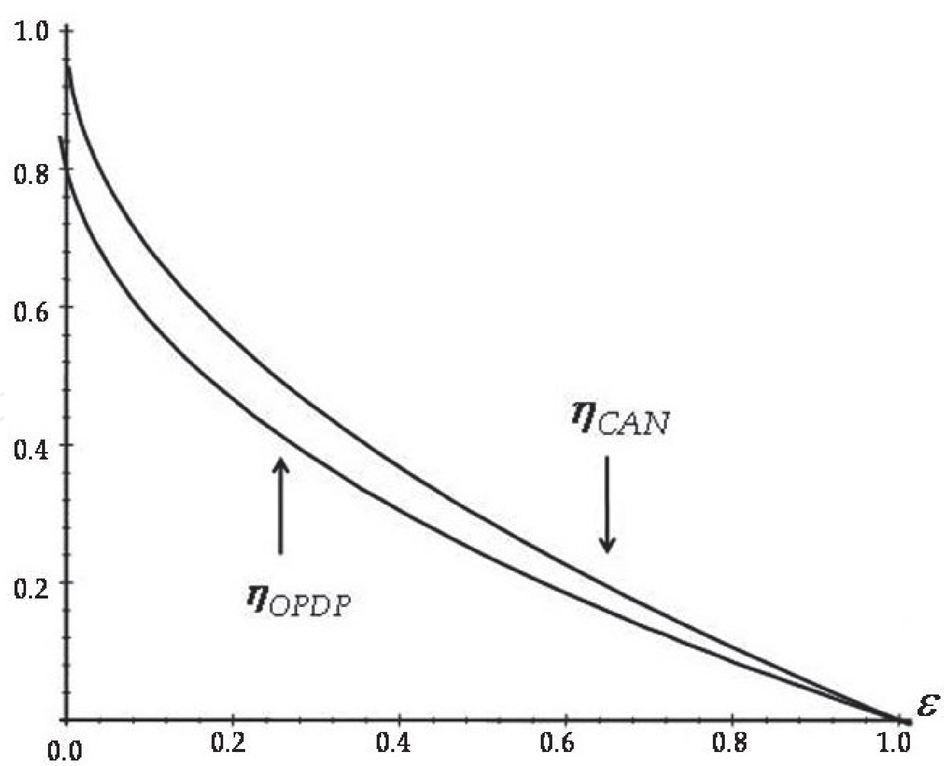

Fig. 7. Comparison between η_{OPDP} obtained here and η_{CAN}, in real plants.

Power plant	T_C	T_H	η_{CAN}	η_{OPDP}	η_{OBS}
Steam power plant, West Thurrock , U K	298	838	0.40367	0.33577	0.360
Geothermal steam plant, Lardarello, Italy	353	523	0.17845	0.1453	0.160
Steam power plant, U S A	298	923	0.43179	0.36006	0.400
Combined cycle plant (steam-mercury), U S A	298	783	0.38308	0.31804	0.340

Table 3. Comparison of Curzon and Ahlborn and observed efficiencies with the here approximated obtained efficiency.

Now assuming that z obtained from equation (65) can be expressed as a power series in the parameter λ, we have the following expression for η_{PDP},

$$\eta_{PDP} = 1 - z_{PDP}(\lambda,\varepsilon) = 1 - z_{OPDP}[1 + B_1(\varepsilon)\lambda + B_2(\varepsilon)\lambda^2 + O(\lambda^3)\,. \tag{68}$$

We can find the coefficients B_j, $j = 1,2,....etc.,$ through successive derivatives respect to λ. The two first ones coefficients are:

$$B_1(\varepsilon) = \frac{16(1 - z_{OPDP})(\varepsilon - z_{OPDP})}{z_{OPDP}(5 - 4\varepsilon - 40z_{OPDP})} \tag{69}$$

and

$$B_2 = \frac{4(z_{OPDP}-1)(z_{OPDP}-\varepsilon)}{(1+9\varepsilon-10z_{OPDP})^2}\left\{\frac{[(1-\varepsilon+10z_{OPDP})\ln z_{OPDP} + 8z_{OPDP} - 4\varepsilon - 4](\varepsilon + 1 - 10z_{OPDP})}{1+9\varepsilon-10z_{OPDP}}\times\right.$$

$$\left.\frac{40(z_{OPDP}-1)(z_{OPDP}-\varepsilon)}{1+9\varepsilon-10z_{OPDP}} - \left[(9\varepsilon - 1 - 10z_{OPDP})\ln z_{OPDP} + 4 + 4\varepsilon - 8z_{OPDP}\right]\right\} \tag{70}$$

which are positive for ε values in the interval $0 < \varepsilon < 1$, as we can see in Figure 8.

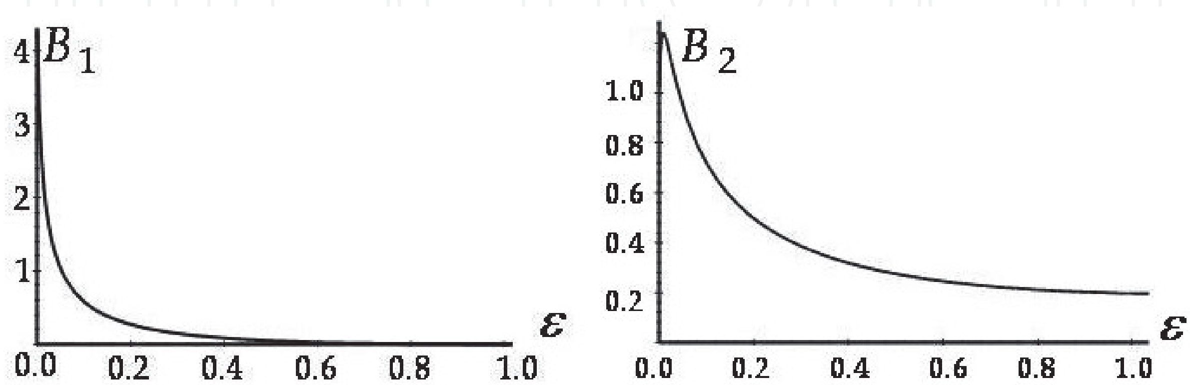

Fig. 8. First and second order coefficients, $B_1 = B_1(\varepsilon)$, $B_2 = B_2(\varepsilon)$, of (2.21) for $0 < \varepsilon < 1$.

3.2 Ecological efficiency

Now we consider the entropy production given by

$$\sigma \equiv \frac{\Delta S}{t_{tot}} = \frac{1}{t_{tot}}(-\frac{Q_H}{T_H} + \frac{Q_C}{T_C}) \tag{71}$$

which becomes,

$$\sigma = \frac{\alpha T_H^k (1+\lambda \ln z)(z-\varepsilon)}{T_C(\frac{1}{(1-u)^k} + \frac{z}{(zu-\varepsilon)^k})}; \tag{72}$$

so that the ecological function for Curzon and Ahlborn engine takes the form,

$$E(x,z) = \frac{T_H^k \alpha (1+\lambda \ln z)(1+\varepsilon-2z)}{\frac{1}{(1-u)^k} + \frac{z}{(zu-\varepsilon)^k}}. \tag{73}$$

As in the previous sections we find the function $z(\varepsilon)$ that follows from the maximization of function $E(u,z)$, which permits obtain the corresponding efficiency for the value $k = 5/4$, namely, the Dulong and Petit heat transfer law, previously defined. Upon setting $\partial E / \partial u = 0$ and $\partial E / \partial z = 0$, we obtain from the first condition that

$$u = \frac{z^{\frac{2}{k+1}} + \varepsilon}{z + z^{\frac{2}{k+1}}}, \tag{74}$$

and from the second one,

$$\frac{((1+\varepsilon-2z)\lambda - 2z(1+\lambda \ln z))(zu-\varepsilon)}{(1+\lambda \ln z)(1+\varepsilon-2z)(zu-\varepsilon-kuz)z} - \frac{(1-u)^k}{(zu-\varepsilon)^k + z(1-u)^k} = 0. \tag{75}$$

Substituting now Equation (74) for u in Equation (75) we are led to the following expression,

$$(-2(1+\lambda \ln z)z + (1+\varepsilon-2z)\lambda)(z^2 + z^{\frac{k+3}{k+1}})(z-\varepsilon) = z(1+\lambda \ln z)(1+\varepsilon-2z)(z^{\frac{k+3}{k+1}} - \varepsilon z^{\frac{2}{k+1}} - (z^{\frac{k+3}{k+1}} + z\varepsilon)k). \tag{76}$$

The analytical solution of Eq. (76) is not feasible when the exponents of z are not integers, which is the present case, because with $k = 5/4$, Dulong and Petit's heat transfer law, one has $(k+3)/(k+1) = 17/9$ and $2/(k+1) = 8/9$.

The numerical solution of Eq. (76) shows that any solution falls into the region bounded by solutions for $\lambda = 0$ and $\lambda = 1$, (Ladino-Luna, 2008). It can be appreciated that within the interval $0 \le \varepsilon \le 1$, which is the only one physically relevant, the curve (76) can be fitted with a parabolic curve. The simplest approximation that allows for a parabolic fit for $0 \le \lambda \le 1$ is the following modification of the exponents:

$$\frac{k+3}{k+1} \sim 2, \quad \frac{2}{k+1} \sim 1. \tag{77}$$

These approximations allow the following approximate analytical expression for $z(\varepsilon,\lambda)$

$$2(-2(1+\lambda\ln z)z+(1+\varepsilon-2z)\lambda)(z-\varepsilon)-(1+\lambda\ln z)(1+\varepsilon-2z)((z-\varepsilon)-(z+\varepsilon)k)=0. \tag{78}$$

For the case $\lambda=0$, that corresponds to instantaneous adiabats, and taking $k=5/4$ in Equation (78), the value of the positive root $z_{OEDP}(\varepsilon)$ is obtained,

$$z_{OEDP}=\frac{1-\varepsilon+\sqrt{649\varepsilon^2+646\varepsilon+1}}{36}. \tag{79}$$

The negative root has no physical meaning because efficiencies must always be positive. Figure 9 shows a comparison between fitted numerical values of η_{MEDP} (Angulo-Brown & Páez-Hernández, 1993; Árias-Hernández & Angulo-brown, 1994) and $\eta_{OEDP}=1-z_{ODP}$.

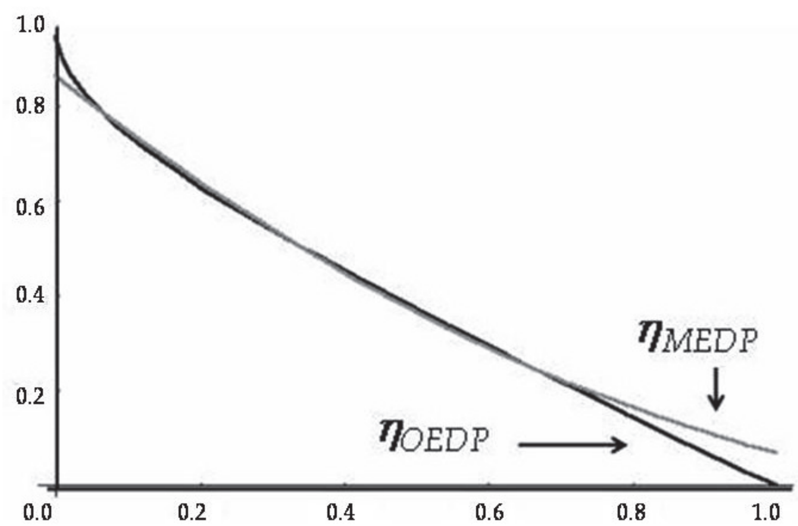

Fig. 9. Approximated Ecological efficiency η_{OEDP}, compared with a fitted of η_{MEDP}.

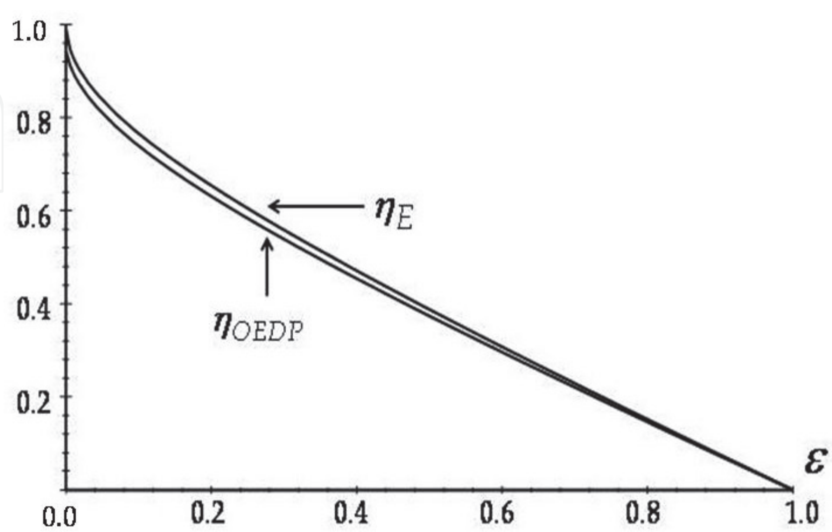

Fig. 10. Ecological efficiency for Newton's heat transfer and Dulong-Petit's heat transfer.

Notice that η_{OEDP} is a better result from a theoretical point of view, because it goes to zero as $\varepsilon \to 1$ as it should be. Figure 10 shows the comparison between η_E and η_{OEDP} where $\eta_{OEDP} < \eta_E$.
Let it be assumed now that z given by equation (78) is a power series in the parameter λ, i.e.,

$$\eta_{EDP} = 1 - z_{EDP}(\lambda,\varepsilon) = 1 - z_{EDP}(1 + b_1(\varepsilon)\lambda + b_2(\varepsilon)\lambda^2 + O(\lambda^3)), \tag{80}$$

and let us proceed to the calculation of the coefficients of the powers in λ. To this end one takes $z_0 = z_{EDP}(\varepsilon, \lambda = 0)$ and from (78) the coefficients are calculated by successively taking the derivative with respect to λ and evaluating at $\lambda = 0$. The first two are:

$$b_1(\varepsilon) = \frac{-2z_0 + 2\varepsilon - 6z_0\varepsilon + 2\varepsilon^2 + 4z_0^2}{z_0(-9z_0 - \frac{1}{4}\varepsilon + \frac{1}{4})} \tag{81}$$

and

$$b_2(\varepsilon) = \frac{1}{2z_0}(A_1(\varepsilon) + A_2(\varepsilon)), \tag{82}$$

where

$$A_1(\varepsilon) = \frac{4b_1}{(1-\varepsilon-20z_0)^2}\Big\{-160(z_0-\varepsilon)(1+\varepsilon-2z_0) + \Big[(-36z_0 + 9\varepsilon + 1)\ln z_0 - 18z_0$$

$$+9\varepsilon + 1 + \frac{9(\varepsilon^2+\varepsilon)}{z_0} + 8(1+\varepsilon-2z_0) - 16(z_0-\varepsilon)\Big](-1+\eta+20z_0)\Big\}, \tag{83}$$

and

$$A_2(\varepsilon) = -\frac{8(z_0-\varepsilon)(1+\varepsilon-2z_0)}{(1-\varepsilon-20z_0)^2}\left[(36z_0+\varepsilon-1)\ln z_0 + 48z_0 - 40\varepsilon - 8 + \frac{1+\varepsilon-2z_0}{z_0}(z_0+9\varepsilon)\right] \tag{84}$$

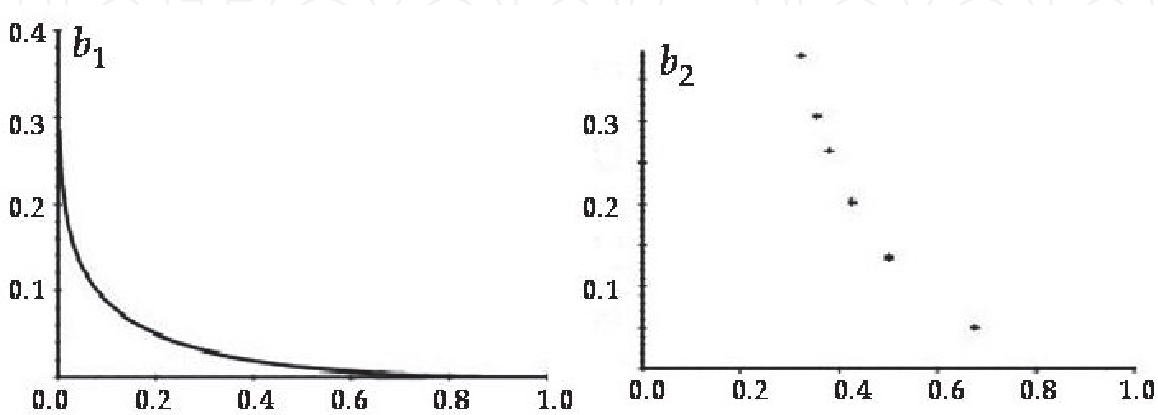

Fig. 11. First and second order coefficients $b_1 = b_1(\varepsilon)$ and $b_2 = b_2(\varepsilon)$ in (80)

To assume that (80) is valid it requires that $\eta_{EDP} \leq 1$. For this to be so, b_1 and b_2 must to be positive when $0 < \varepsilon < 1$ and when $\lambda < 1$, i.e. there must exist an interval for ε into which the coefficients have positive values, near to zero. Figure 11 shows that in fact this is correct. This guarantees that Eq. (80) is valid and that $1 - z_0$ is an upper bound of η_{DP}, but not the upper bound η_C.

4. The van der Waals gas

The internal energy in the case of a van der Waals gas for n moles, with a change of temperature $\Delta T = T - T_0$, at volume V, and with the characteristic constant a of the system, and the constant heat capacity C can be written as,

$$U = nC(T - T_0) - \frac{an^2}{V}. \tag{85}$$

So that taking the temporary derivative for an adiabatic process,

$$\frac{dU}{dt} = \frac{an^2}{V^2} \cdot \frac{dV}{dt}, \tag{86}$$

the first law of thermodynamics leads to

$$\frac{dU}{dt} = \frac{dQ}{dt} - p_{ext}\frac{dV}{dt} - (p - p_{ext})\frac{dV}{dt}, \tag{87}$$

taking p as the internal pressure and p_{ext} as the pressure of surroundings. Combining Equations (86) and Eq. (87), in mechanical equilibrium, we obtain

$$\frac{an^2}{V^2}\frac{dV}{dt} = \frac{dQ}{dt} - p\frac{dV}{dt}, \tag{88}$$

so that, for a non linear heat transfer law, more general than Dulong and Petit heat transfer law, as

$$\frac{dQ}{dt} = \alpha(T - T_0)^k, \tag{89}$$

with the constant thermal conductance α, and the constant exponent k, $k > 1$, from Equation. (88), in an isothermal process,

$$(\frac{an^2}{V^2} + p)\frac{dV}{dt} = \alpha(T - T_0)^k \cdot \tag{90}$$

On other hand, the state equation for a van der Waals gas, with a constant b characteristic of the system, which is a more realistic model for a real gas, takes the following expression, with constant parameters a and b,

$$p = \frac{nRT}{V - nb} - \frac{an^2}{V^2}, \tag{91}$$

whose derivative respect T at $p = constant$ leads to

$$0 = \frac{nR}{V - nb} + nRT\left(-\frac{1}{(V - nb)^2}\frac{\partial V}{\partial T} + \frac{2a}{V^2}\frac{\partial V}{\partial T} \right). \tag{92}$$

By taking $n = 1$, Equation (91) into Equation (90) leads to

$$\frac{RT_0}{V - b}\frac{dV}{dt} = \alpha(T - T_0)^k \cdot \tag{93}$$

In the case of a Curzon and Ahlborn cycle (Figure 1), for the heat exchange between the engine and the reservoirs, Equation (93) leads to the time of the isothermal processes by taking its integration. Moreover in the case of adiabatic processes $dQ / dt = 0$, so that Equation (87) reduces to

$$\frac{dU}{dt} = -p\frac{dV}{dt}, \tag{94}$$

and one can obtain

$$C_V \ln T = -R\ln(V - b), \tag{95}$$

or as it is usually written,

$$T(V - b)^{\frac{R}{C_V}} = constant. \tag{96}$$

Also, the duration time of the adiabatic processes can be obtained by integration of (93). Therefore the duration time of all processes in the cycle can be obtained from Equation (94), and Equation (96) leads to the relation between temperatures of the engine and the changes of volume in the adiabatic transformation.

4.1 Power output and ecological function

Taking into account the difference of temperatures between the engine and its reservoirs (Figure 1), it can be written the time for all of the processes in the cycle from Equation (93) For the isothermal processes, Eq. (93) can be written as

$$\frac{RT_{HW}}{V - b}\cdot\frac{dV}{dt} = \alpha(T_H - T_{HW})^k, \quad \text{and} \quad \frac{RT_{CW}}{V - b}\cdot\frac{dV}{dt} = \alpha(T_{CW} - T_C)^k, \tag{97}$$

and by direct integration of Equationss (97) we obtain

$$t_1 = \frac{RT_{HW}}{\alpha(T_H - T_{HW})^k}\ln\frac{V_2 - b}{V_1 - b}, \quad \text{and} \quad t_3 = \frac{RT_{CW}}{\alpha(T_{CW} - T_C)^k}\ln\frac{V_4 - b}{V_3 - b}, \tag{98}$$

Analogously, the time for the adiabatic processes can be obtained as

$$t_2 = \frac{RT_{HW}}{\alpha(T_H - T_{HW})^k}\ln\frac{V_3 - b}{V_2 - b}, \quad \text{and} \quad t_4 = \frac{RT_{CW}}{\alpha(T_{CW} - T_C)^k}\ln\frac{V_1 - b}{V_4 - b}, \tag{99}$$

Now, taking into account Equation (96), for the first adiabatic process,

$$T_{HW}(V_2-b)^{\frac{R}{C_V}}=T_{CW}(V_3-b)^{\frac{R}{C_V}}\,,\quad \text{or,}\quad \ln\frac{V_3-b}{V_2-b}=\frac{C_V}{R}\ln\frac{T_{HW}}{T_{CW}}\,, \tag{100}$$

and for the second adiabatic process,

$$T_{CW}(V_4-b)^{\frac{R}{C_V}}=T_{HW}(V_1-b)^{\frac{R}{C_V}}\,,\quad \text{or,}\quad \ln\frac{V_1-b}{V_4-b}=\frac{C_V}{R}\ln\frac{T_{CW}}{T_{HW}}\,, \tag{101}$$

and the combination of Eqs. (100) and (101) allows to obtain the relation

$$\frac{V_3-b}{V_2-b}=\frac{V_4-b}{V_1-b},\quad \text{or,}\quad \frac{V_3-b}{V_4-b}=\frac{V_2-b}{V_1-b} \tag{102}$$

The power output P can be written simplyfing with the same used parameters as,

$$P=\frac{\alpha T_H^k(1-z)\left[\lambda_{VW}\ln z+1\right]}{\frac{1}{(1-u)^k}+\frac{z}{(zu-\varepsilon)^k}}\,, \tag{103}$$

where $\lambda_{VW}=\left[(\gamma-1)\ln\frac{V_3-b}{V_1-b}\right]^{-1}$. One can see that $b\to 0$ leads to $\lambda_{VW}\to\lambda$ in Equation (29), and one can see that $b\to 0$ and $k\to 1$ reduce (103) to expression of P, such as it was found previously (Ladino-Luna, 2002, 2005). An expression of power series in λ_{VW} leads to the efficiency that can be obtained following the procedure in those references.

In the case of ecological function it is necessary to build the entropy production σ, $\sigma=\frac{\Delta S}{t_{tot}}$, so that (33) can be written since

$$\Delta S=\Delta S_{1\to 2}+\Delta S_{3\to 4}\,, \tag{104}$$

where $\Delta S_{1\to 2}$ is the change of entropy in the first isothermal branch and $\Delta S_{3\to 4}$ is the change of entropy at the second isothermal branch. For heat reservoirs, $\Delta S=\frac{Q}{T}$, assumed as it is only in the transfer processes between the reservoirs and the engine,

$$\Delta S_{1\to 2}=\frac{Q_{1\to 2}}{T_H}=R\frac{T_{HW}}{T_H}\ln\frac{V_2-b}{V_1-b}\quad \text{and}\quad \Delta S_{3\to 4}=\frac{Q_{3\to 4}}{T_C}=R\frac{T_{CW}}{T_C}\ln\frac{V_4-b}{V_3-b}\,; \tag{105}$$

so that Eq. (104) can be written as

$$\Delta S=R\left[\frac{T_{HW}}{T_H}\ln\frac{V_2-b}{V_1-b}-\frac{T_{CW}}{T_C}\ln\frac{V_3-b}{V_4-b}\right], \tag{106}$$

and by using (102) and (96) one can obtain the entropy production as,

$$\sigma=\frac{\alpha T_H^k(\varepsilon-z)\left[\lambda_{VW}\ln z+1\right]}{\frac{1}{(1-u)^k}+\frac{z}{(zu-\varepsilon)^k}}\,, \tag{107}$$

then, by using (3), (98) and (99) the ecological function can be written as

$$E = \frac{\alpha T_1^k (1 - 2z + \varepsilon)\left[\lambda_{VW} \ln z + 1\right]}{\frac{1}{(1-u)^k} + \frac{z}{(zu-\varepsilon)^k}} . \tag{108}$$

One can see that the structure of Eq. (108) leads to the case with Newton's heat transfer law when the limit $k \to 1$ is. It is also obtained the case of Newton heat transfer with an ideal gas as the working substance when $k = 1$ and $b = 0$. A general form of ecological function and power output function can be obtained by replacing λ_{VW} instead of λ, and with approximations for the cases when $k > 1$. z_{EDP} and η_{EDP} (Ladino-Luna, 2008) are modified with the substitution $V - b$ instead of V.

The corresponding maximization of ecological function taking Dulong-Petit's heat transfer and a van der Waals gas as the working substance can be found with the substitution λ_{VW} instead of λ in all of the process to build the ecological efficiency. In the case of power output with the same substitution, we obtain the approximate formula for the efficiency when λ_{VW} goes to zero, and a similar power series of the efficiency as a function of λ_{VW},

$$\eta_{PDPVW} = 1 - z_{PDP}(\lambda_{VW}, \varepsilon) = 1 - z_{OPDP}(1 + b_1(\varepsilon)\lambda_{VW} + b_2(\varepsilon)\lambda_{VW}^2 + O(\lambda_{VW}^3)) , \tag{109}$$

wher z_{OPDP} is the same approximate efficiency previously found in section 3, following the procedure by Ladino-Luna (2003). At the limit $\lambda_{VW} \to 0$ we obtain $\eta_{PDPVW}(\lambda_{VW} = 0) = \eta_{OPDP}$ where η_{OPDP} is the same approximate efficiency found in Section 3. As one can see, $\eta_{PDPVW}(\lambda_{VW} = 0) < \eta_{CAN}$, so η_{CAN} can be consider as an upper bound for the efficiencies that taking into account the time of the adiabatic processes in the Curzon and Ahlborn cycle.

5. Conclusions

A first result is the fact that the efficiency for a Carnot type engine depends on the size of the engine, the compression ratio, as represented by the parameter $\lambda \sim \left[\ln(V_3 / V_1)\right]^{-1}$ or $\lambda_{VW} = \left[(\gamma - 1) \ln \frac{V_3 - b}{V_1 - b}\right]^{-1}$. Leading term in power series corresponds to the exact value numerically calculated without explicitly taking into account the dependence on λ, and is an upper bound for the value of the efficiency η_{DP}; in fact the larger the ratio V_3 / V_1 (or $(V_3 - b)/(V_1 - b)$), the larger the efficiency becomes. The comparison between the upper bound of the efficiency calculated with the proposed approximations and a fitted curve obtained of the numerical values from cited references shows the goodness of the made approximations in case of $k = 5/4$. It is worthwhile mentioning that exist an interval for ε, ε ~0.5, were the approximation employed is acceptable within 5% of the true value of $z(\varepsilon)$ for $0 \le \lambda \le 1$ as shown. A last result is shown in Figures where one can appreciate that the difference between using Newton's or Dulong-Petit's heat transfer laws does not lead to an important difference in the value of the ecological efficiency. It has also been shown that for the Dulong-Petit heat transfer law and the ideal gas law, the limit $\lambda \to 0$ reduces to the reported result. Also, the results suggest that can be extended a new interpretation as the way to real performance of the thermal plants. It shows a mixture between Newton and Dulong-Petit heat transfer laws. Non-endoreversible cycles could be analyzed using non-instantaneous adiabats together with non-linear heat transfer .

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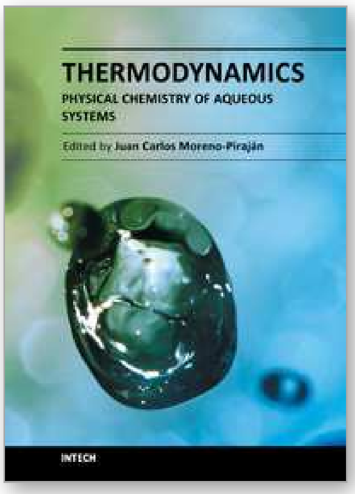

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