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Efficient Nonlinear Model Predictive Control for Affine System

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1. Introduction

Model predictive control (MPC) refers to the class of computer control algorithms in which a dynamic process model is used to predict and optimize process performance. Since its lower request of modeling accuracy and robustness to complicated process plants, MPC for linear systems has been widely accepted in the process industry and many other fields. But for highly nonlinear processes, or for some moderately nonlinear processes with large operating regions, linear MPC is often inefficient. To solve these difficulties, nonlinear model predictive control (NMPC) attracted increasing attention over the past decade (Qin *et al.*, 2003, Cannon, 2004). Nowadays, the research on NMPC mainly focuses on its theoretical characters, such as stability, robustness and so on, while the computational method of NMPC is ignored in some extent. The fact mentioned above is one of the most serious reasons that obstruct the practical implementations of NMPC.

Analyzing the computational problem of NMPC, the direct incorporation of a nonlinear process into the linear MPC formulation structure may result in a non-convex nonlinear programming problem, which needs to be solved under strict sampling time constraints and has been proved as an NP-hard problem (Zheng, 1997). In general, since there is no accurate analytical solution to most kinds of nonlinear programming problem, we usually have to use numerical methods such as Sequential Quadratic Programming (SQP) (Ferreau *et al.*, 2006) or Genetic Algorithm (GA) (Yuzgec *et al.*, 2006). Moreover, the computational load of NMPC using numerical methods is also much heavier than that of linear MPC, and it would even increase exponentially when the predictive horizon length increases. All of these facts lead us to develop a novel NMPC with analytical solution and little computational load in this chapter.

Since affine nonlinear system can represent a lot of practical plants in industry control, including the water-tank system that we used to carry out the simulations and experiments, it has been chosen for propose our novel NMPC algorithm. Follow the steps of research work, the chapter is arranged as follows:

In Section 2, analytical one-step NMPC for affine nonlinear system will be introduced at first, then, after description of the control problem of a water-tank system, simulations will be carried out to verify the result of theoretical research. Error analysis and feedback compensation will be discussed with theoretical analysis, simulations and experiment at last.

Then, in Section 3, by substituting reference trajectory for predicted state with stair-like control strategy, and using sequential one-step predictions instead of the multi-step

prediction, the analytical multi-step NMPC for affine nonlinear system will be proposed. Simulative and experimental control results will also indicate the efficiency of it. The feedback compensation mentioned in Section 2 is also used to guarantee the robustness to model mismatch.

Conclusion and further research direction will be given at last in Section 4.

2. One-step NMPC for affine system

2.1 Description of NMPC for affine system

Consider a time-invariant, discrete, affine nonlinear system with integer k representing the current discrete time event:

$$x_{k+1} = f(x_k) + g(x_k) \times u_k + \xi_k \quad (1a)$$

$$\text{s. t. } x_k \in X \subseteq \mathbb{R}^n \quad (1b)$$

$$u_k \in U \subseteq \mathbb{R}^m \quad (1c)$$

$$\xi_k \in \mathbb{R}^n \quad (1d)$$

In the above, u_k, x_k, ξ_k are input, state and disturbance of the system respectively, $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$, $g: \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m}$ are corresponding nonlinear mapping functions with proper dimension.

Assume $\hat{x}_{k+j|k}$ are predictive values of x_{k+j} at time k , $\Delta u_k = u_k - u_{k-1}$ and $\Delta \hat{u}_{k+j|k}$ are the solutions of future increment of u_{k+j} at time k , then the objective function J_k can be written as follow:

$$J_k = F(\hat{x}_{k+p|k}) + \sum_{j=0}^{p-1} G(\hat{x}_{k+j|k}, \Delta u_{k+j|k}) \quad (2)$$

The function $F(\cdot)$ and $G(\cdot, \cdot)$ represent the terminal state penalty and the stage cost respectively, where p is the predictive horizon.

In general, J_k usually has a quadratic form. Assume $w_{k+j|k}$ is the reference value of x_{k+j} at time k which is called reference trajectory (the form of $w_{k+j|k}$ will be introduced with detail in Section 2.2 and 3.1 for one-step NMPC and multi-step NMPC respectively), semi-positive definite matrix Q and positive definite matrix R are weighting matrices, (2) now can be written as :

$$J_k = \sum_{j=1}^p \left\| \hat{x}_{k+j|k} - w_{k+j|k} \right\|_Q^2 + \sum_{j=0}^{p-1} \left\| \Delta u_{k+j|k} \right\|_R^2 \quad (3)$$

Corresponding to (1) and (3), the NMPC for affine system at each sampling time now is formulated as the minimization of J_k , by choosing the increments sequence of future control input $[\Delta u_{k|k} \quad \Delta u_{k+1|k} \quad \cdots \quad \Delta u_{k+p-1|k}]$, under constraints (1b) and (1c).

By the way, for simplicity, In (3), part of J_k is about the system state x_k , if the output of the system $y_k = Cx_k$, which is a linear combination of the state (C is a linear matrix), we can rewrite (3) as follow to make an objective function J_k about system output:

$$J_k = \sum_{j=1}^p \left\| C\hat{x}_{k+j|k} - w_{k+j|k} \right\|_Q^2 + \sum_{j=0}^{p-1} \left\| \Delta u_{k+j|k} \right\|_R^2 = \sum_{j=1}^p \left\| \hat{y}_{k+j|k} - w_{k+j|k} \right\|_Q^2 + \sum_{j=0}^{p-1} \left\| \Delta u_{k+j|k} \right\|_R^2 \quad (4)$$

And sometimes, $\Delta u_{k+j|k}$ in J_k could also be changed as $u_{k+j|k}$ to meet the need of practical control problems.

2.2 One-step NMPC for affine system

Except for some special model, such as Hammerstein model, analytic solution of multi-step NMPC could not be obtained for most nonlinear systems, including the NMPC for affine system mentioned above in Section 2.1. But if the analytic inverse of system function exists (could be either state-space model or input-state model), the one-step NMPC always has the analytic solution. So all the research in this chapter is not only suitable for affine nonlinear system, but also suitable for other nonlinear systems, that have analytic inverse system function.

Consider system described by (1a-1d) again, the one-step prediction can be deduced directly as follow with only one unknown data $\Delta u_{k|k} = u_{k|k} - u_{k-1}$ at time k :

$$\hat{x}_{k+1|k} = f(x_k) + g(x_k) \cdot u_{k|k} = f(x_k) + g(x_k) \cdot u_{k-1} + g(x_k) \cdot \Delta u_{k|k} = \hat{x}_{k+1|k}^1 + g(x_k) \cdot \Delta u_{k|k} \quad (5)$$

In (5), $\hat{x}_{k+1|k}^1$ means the part which contains only known data (x_k and u_{k-1}) at time k , and $g(x_k) \cdot \Delta u_{k|k}$ is the unknown part of predictive state $\hat{x}_{k+1|k}$.

If there is no model mismatch, the predictive error of (5) will be $\tilde{x}_{k+1|k} = x_{k+1} - \hat{x}_{k+1|k} = \xi_{k+1}$.

Especially, if ξ_k is a stationary stochastic noise with zero mean and variance $E[\xi_k] = \delta^2$, it is easy known that $E[\tilde{x}_{k+1|k}] = 0$, and $E[(\tilde{x}_{k+1|k} - E[\tilde{x}_{k+1|k}])^T \cdot (\tilde{x}_{k+1|k} - E[\tilde{x}_{k+1|k}])] = n\delta^2$, in another word, both the mean and the variance of the predictive error have a minimum value, so the prediction is an optimal prediction here in (5).

Then if the setpoint is x_{sp} , and to soften the future state curve, the expected state value at time $k+1$ is chosen as $w_{k+1|k} = \alpha x_k + (1 - \alpha)x_{sp}$, where $\alpha \in [0, 1)$ is called soften factor, thus the objective function of one-step NMPC can be written as follow:

$$J_k = \left\| \hat{x}_{k+1|k} - w_{k+1|k} \right\|_Q^2 + \left\| \Delta u_{k|k} \right\|_R^2 \quad (6)$$

To minimize J_k without constraints (1b) and (1c), we just need to have $\frac{\partial J_k}{\partial \Delta u_{k|k}} = 0$ and

$\frac{\partial^2 J_k}{\partial \Delta u_{k|k}^2} > 0$, then:

$$\Delta u_{k|k} = -(g(x_k)^T \cdot Q \cdot g(x_k) + R)^{-1} \cdot (g(x_k) \cdot Q \cdot (\hat{x}_{k+1|k}^1 - w_{k+1|k})) \quad (7)$$

Mark $H = g(x_k)^T \cdot Q \cdot g(x_k) + R$ and $F = g(x_k) \cdot Q \cdot (w_{k+1|k} - \hat{x}_{k+1|k}^1)$, so the increment of instant future input is:

$$\Delta u_{k|k} = -H^{-1}F \quad (8)$$

But in practical control problem, limitations on input and output always exist, so the result of (8) is usually not efficient. To satisfy the constraints, we can just put logical limitation on amplitudes of u_k and x_k , or some classical methods such as Lagrange method could be used. For simplicity, we only discuss about the Lagrange method here in this chapter.

First, suppose every constraint in (1b) and (1c) could be rewritten in the form as $a_i^T \Delta u_{k|k} \leq b_i$, $i = 1, 2, \dots, q$, then the matrix form of all constraints is:

$$A \Delta u_{k|k} \leq B \quad (9)$$

In which, $A = [a_1^T \ a_2^T \ \dots \ a_q^T]^T$ $B = [b_1 \ b_2 \ \dots \ b_q]^T$.

Choose Lagrange function as $L_k(\lambda_i) = J_k + \lambda_i^T (a_i^T \Delta u_{k|k} - b_i)$, $i = 1, 2, \dots, q$, let

$\frac{\partial L}{\partial \Delta u_{k|k}} = H \Delta u_{k|k} + F + a_i \lambda_i = 0$ and $\frac{\partial L}{\partial \lambda_i} = a_i^T \Delta u_{k|k} - b_i = 0$, then:

$$\Delta u_{k|k} = -H^{-1}(F + a_i \lambda_i) \quad (10a)$$

$$\lambda_i = -\frac{a_i^T H^{-1} F + b_i}{a_i^T H^{-1} a_i} \quad (10b)$$

If $\lambda_i \leq 0$ in (10b), means that the corresponding constraint has no effect on $\Delta u_{k|k}$, we can choose $\bar{\lambda}_i = 0$, but if $\lambda_i > 0$ in (10b), the corresponding constraint has effect on $\Delta u_{k|k}$ indeed, so we must choose $\bar{\lambda}_i = \lambda_i$, finally, the solution of one-step NMPC with constraints could be:

$$\Delta u_{k|k} = -H^{-1}(F + A^T \bar{\Lambda}) \quad (11)$$

In which, $\bar{\Lambda} = [\bar{\lambda}_1 \ \bar{\lambda}_2 \ \dots \ \bar{\lambda}_q]^T$

2.3 Control problem of the water-tank system

Our plant of simulations and experiments in this chapter is a water-tank control system as that in Fig. 1. and Fig. 2. (We just used one water-tank of this three-tank system). Its affine nonlinear model is achieved by mechanism modeling (Chen *et al.*, 2006), in which the variables are normalized, and the sample time is 1 second here:

$$x_{k+1} = x_k - 0.2021\sqrt{x_k} + 0.01923u_k \quad (12a)$$

$$\text{s. t. } x_k \in [0\%, 100\%] \quad (12b)$$

$$u_k \in [0\%, 100\%] \quad (12c)$$

In (12), x_k is the height of water in the tank, and u_k is the velocity of water flow into the tank, from pump P_1 and valve V_1 , while valve V_2 is always open. In the control problem of the water-tank, for convenience, we choose the system state as the output, that means $y_k = x_k$, and the system functions are $f(x_k) = x_k - 0.2021\sqrt{x_k}$ and $g(x_k) = 0.01923$.

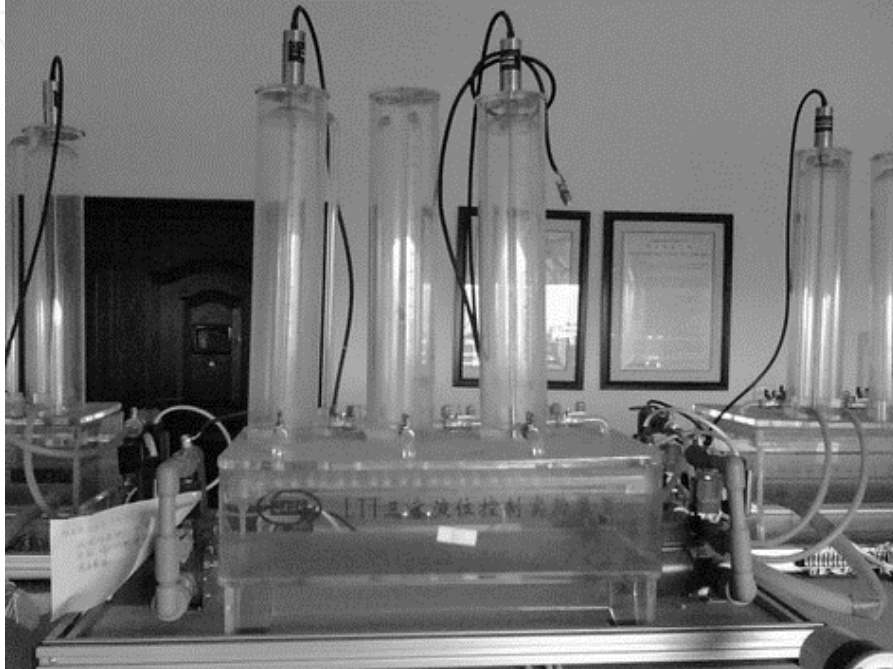


Fig. 1. Photo of the water-tank system

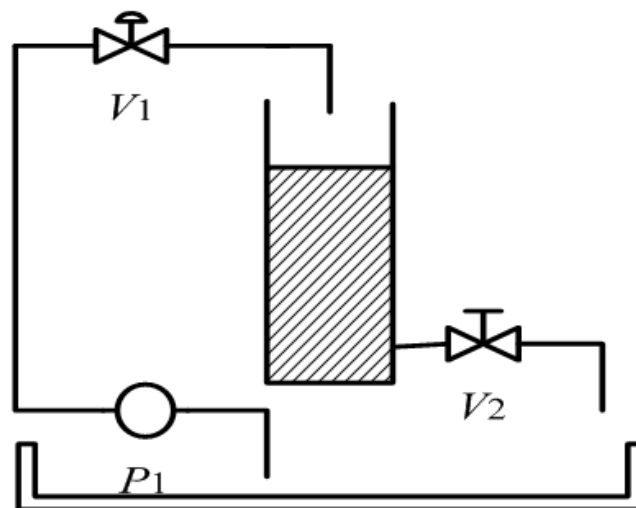


Fig. 2. Structure of the water-tank system

To change the height of the water level, we can change the velocity of input flow, by adjusting control current of valve V_1 , and the normalized relation between the control current and the velocity u_k is shown in Fig. 3.

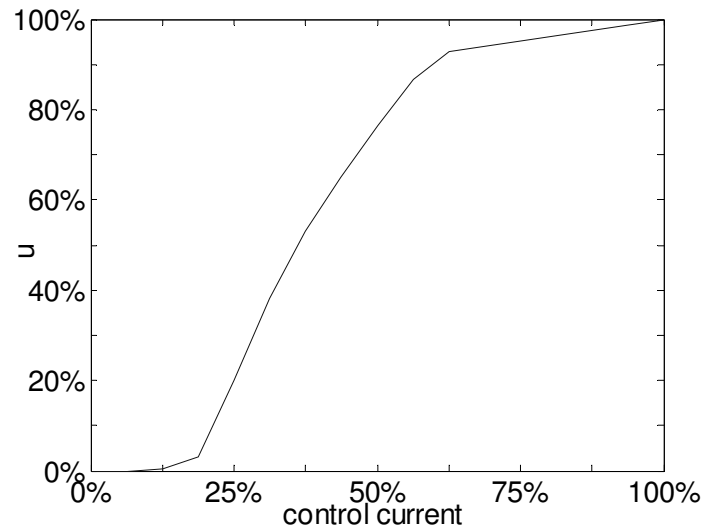


Fig. 3. The relation between control current and input u_k

2.4 One-step NMPC of the water-tank system and its feedback compensation

Choose objective function $J_k = (\hat{x}_{k+1|k} - w_{k+1|k})^2 + 0.001\Delta u_{k|k}^2$, $x_{sp} = 30\%$ and soften factor $\alpha = 0.95$, to carry out all the simulations and the experiment in this section. (except for part of Table 1., where we choose $\alpha = 0.975$)

Suppose there is no model mismatch, the simulative control result of one-step NMPC for water-tank system is obtained as Fig. 4. and it is surely meet the control objective.

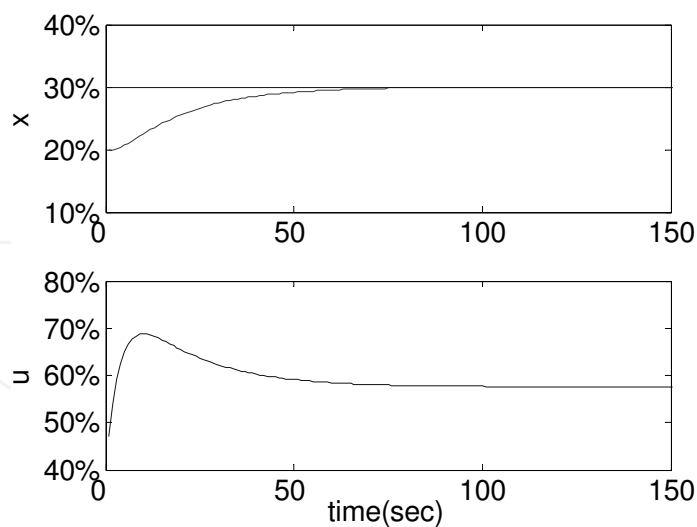


Fig. 4. Simulation of one-step NMPC without model mismatch and feedback compensation

To imitate the model mismatch, we change the simulative model of the plant from $x_{k+1} = x_k - 0.2021\sqrt{x_k} + 0.01923u_k$ to $x_{k+1} = x_k - 110\% \times 0.2021\sqrt{x_k} + 90\% \times 0.01923u_k$, but still use $x_{k+1} = x_k - 0.2021\sqrt{x_k} + 0.01923u_k$ to be the predictive model in one-step NMPC, the result in Fig. 5. now indicates that there is obvious steady-state error.

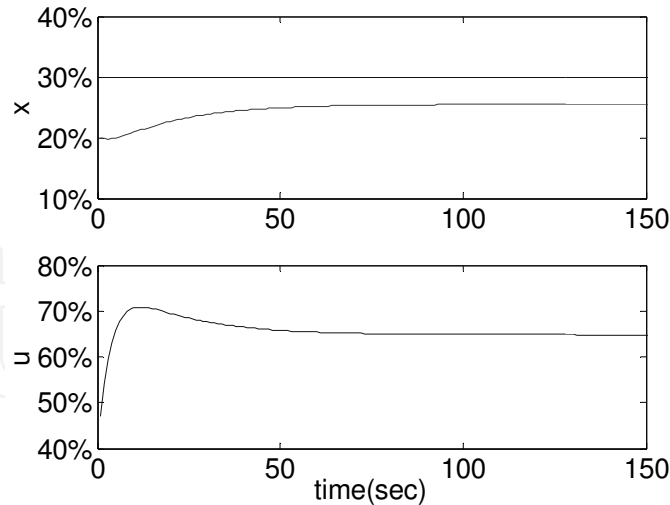


Fig. 5. Simulation of one-step NMPC with model mismatch but without feedback compensation

Proposition 1: For affine nonlinear system $x_{k+1} = f'(x_k) + g'(x_k) \cdot u_k$, if the setpoint is x_{sp} , steady-state is u_s and x_s , and the predictive model is $x_{k+1} = f(x_k) + g(x_k) \cdot u_k$, without consideration of constraints, the steady-state error of one-step NMPC is $e = x_s - x_{sp} = \frac{(f'(x_s) - f(x_s)) + (g'(x_s) - g(x_s)) \cdot u_s}{1 - \alpha}$, in which α is the soften factor.

Proof: If the system is at the steady-state, then we have $x_{k-1} = x_k = x_s$ and $u_{k-1} = u_k = u_s$.

Since $u_{k-1} = u_k = u_s$, so $\Delta u_k = 0$, from (8), we know matrix $F=0$, or equally $(w_{k+1|k} - \hat{x}_{k+1|k}^1) = 0$.

Update the process of one-step NMPC at time k , we have:

$$w_{k+1|k} = \alpha x_k + (1 - \alpha)x_{sp} = \alpha x_s + (1 - \alpha)x_{sp} \quad (13)$$

$$\hat{x}_{k+1|k}^1 = f(x_k) + g(x_k) \cdot u_{k-1} = f(x_s) + g(x_s) \cdot u_s \quad (14)$$

(13)-(14), and notice that $x_s = f'(x_s) + g'(x_s) \cdot u_s$ for steady-state, we get:

$$\begin{aligned} 0 &= \alpha x_s + (1 - \alpha)x_{sp} - f(x_s) - g(x_s) \cdot u_s = x_s - f(x_s) - g(x_s) \cdot u_s + (1 - \alpha)x_{sp} - (1 - \alpha)x_s \\ &= (f'(x_s) - f(x_s)) + (g'(x_s) - g(x_s)) \cdot u_s + (1 - \alpha) \cdot (x_{sp} - x_s) \end{aligned}$$

So:

$$e = x_s - x_{sp} = \frac{(f'(x_s) - f(x_s)) + (g'(x_s) - g(x_s)) \cdot u_s}{1 - \alpha} \quad (15)$$

Proof end.

Because the soften factor $\alpha \in [0, 1)$, thus $1 - \alpha \neq 0$ always holds, the necessary condition for $e = 0$ is $(f'(x_s) - f(x_s)) + (g'(x_s) - g(x_s)) \cdot u_s = 0$. When there is model mismatch, there will be steady-state error, while this error is independent of weight matrix Q and dependent of the

soften factor α . For corresponding discussion on steady-state error of one-step NMPC with constraints, the only difference is (11) will take the place of (8) in the proof.

Table 1. is the comparison on $e = x_s - x_{sp}$ between simulation and theoretical analysis, and they have the same result. (simulative model $x_{k+1} = x_k - 110\% \times 0.2021\sqrt{x_k} + 90\% \times 0.01923u_k$, predictive model $x_{k+1} = x_k - 0.2021\sqrt{x_k} + 0.01923u_k$)

α	Q	$e = x_s - x_{sp}$ Simulation(%)	$e = x_s - x_{sp}$ Value of (15)(%)
0.975	0	-8.3489	-8.3489
	0.001	-8.3489	-8.3489
	0.01	-8.3489	-8.3489
0.95	0	-4.5279	-4.5279
	0.001	-4.5279	-4.5279
	0.01	-4.5279	-4.5279

Table 1. Comparison on $e = x_s - x_{sp}$ between simulation and theoretical analysis

From (15) we know, we cannot eliminate this steady-state error by adjusting α , so feedback compensation could be used here, mark the predictive error e_k at time k as follow:

$$e_k = x_k - \hat{x}_{k|k-1} = x_k - (\hat{x}_{k|k-1}^1 + g(x_{k-1}) \cdot \Delta u_{k-1}) \quad (16)$$

In which, x_k is obtained by system feedback at time k, and $\hat{x}_{k|k-1}$ is the predictive value of x_k at time k-1.

Then add e_k to the predictive value of x_{k+1} at time k directly, so (5) is rewritten as follow:

$$\hat{x}_{k+1|k} = f(x_k) + g(x_k) \cdot u_{k-1} + g(x_k) \cdot \Delta u_{k|k} + e_k = \hat{x}_{k+1|k}^1 + g(x_k) \cdot \Delta u_{k|k} + e_k \quad (17)$$

Use this new predictive value to carry out one-step NMPC, the simulation result in Fig. 6. verify its robustness under model mismatch, since there is no steady-state error with this feedback compensation method.

The direct feedback compensation method above is easy to understand and carry out, but it is very sensitive to noise. Fig. 7. is the simulative result of it when there is noise add to the system state, we can see that the input vibrates so violently, that is not only harmful to the actuator in practical control system, but also harmful to system performance, because the actuator usually cannot always follow the input signal of this kind.

To develop the character of feedback compensation, simply, we can use the weighted average error \bar{e}_k instead of single e_k in (17):

$$\hat{x}_{k+1|k} = \hat{x}_{k+1|k}^1 + g(x_k) \cdot \Delta u_{k|k} + \sum_{i=1}^s h_i \cdot e_{k+1-i} = \hat{x}_{k+1|k}^1 + g(x_k) \cdot \Delta u_{k|k} + \bar{e}_k, \quad \sum_{i=1}^s h_i = 1 \quad (18)$$

Choose $i = 20$, $h_i = 0.05$, the simulative result is shown in Fig. 8. Compared with Fig. 7. it has almost the same control performance, but the input is much more smooth now. Using the same method and parameters, experiment has been done on the water-tank system, the result in Fig. 9. also verifies the efficiency of the proposed one-step NMPC for affine systems with feedback compensation.

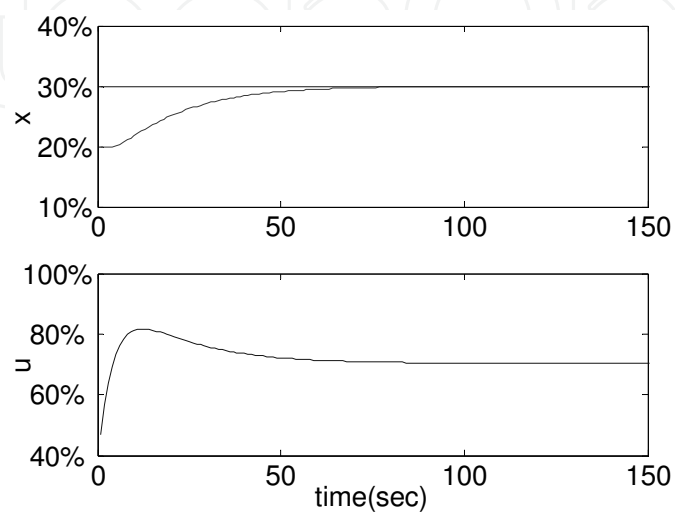


Fig. 6. Simulation of one-step NMPC with model mismatch and direct feedback compensation

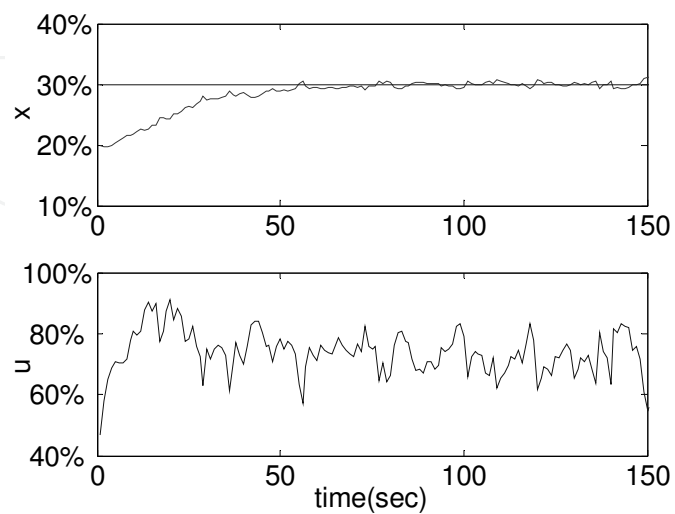


Fig. 7. Simulation of one-step NMPC with model mismatch, noise and direct feedback compensation

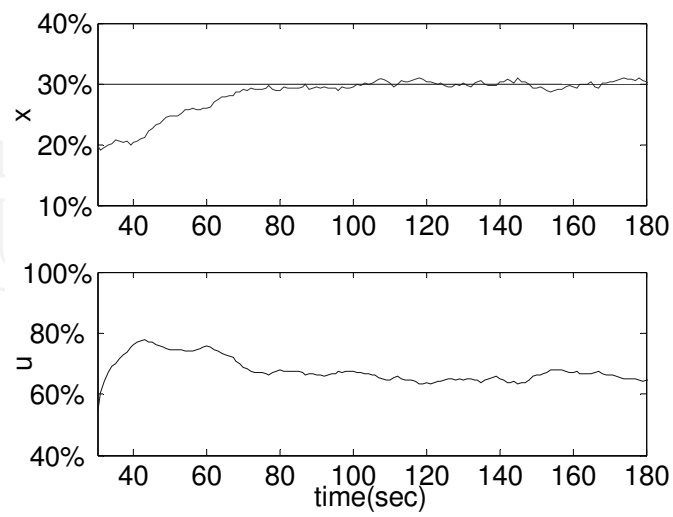


Fig. 8. Simulation of one-step NMPC with model mismatch, noise and smoothed feedback compensation

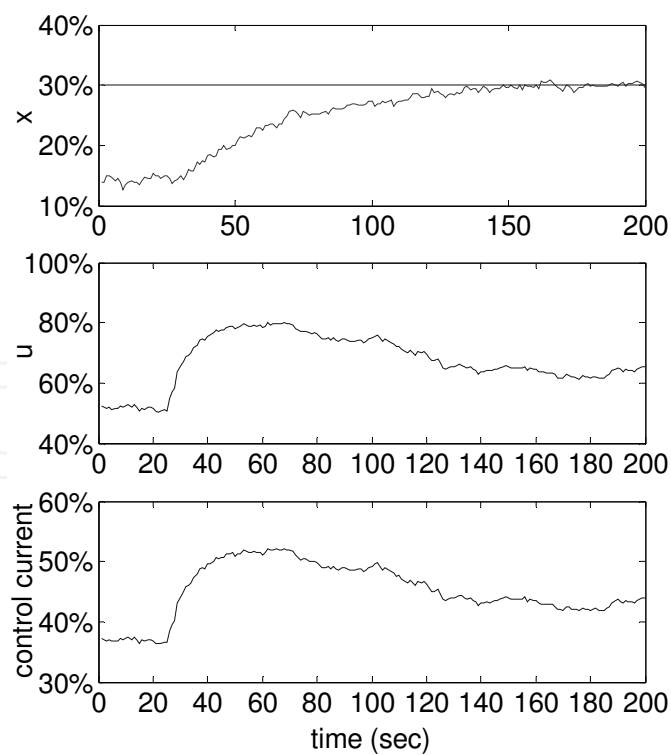


Fig. 9. Experiment of one-step NMPC with setpoint $x_{sp} = 30\%$

3. Efficient multi-step NMPC for affine system

Since reference trajectory and stair-like control strategy will be used to establish efficient multi-step NMPC for affine system in this chapter, we will introduce them in Section 3.1 and 3.2 at first, and then, the multi-step NMPC algorithm will be discussed with theoretical research, simulations and experiments.

3.1 Reference trajectory for future state

In process control, the state usually meets the objective in the form of setpoint along a softer trajectory, rather than reach the setpoint immediately in only one sample time. This may be because of the limit on control input, but a softer change of state is often more beneficial to actuators, even the whole process in practice. This trajectory, usually called reference trajectory, often can be defined as a first order exponential curve:

$$w_{k+j|k} = \alpha w_{k+j-1|k} + (1 - \alpha)x_{sp}, j = 1, 2, \dots, p - 1 \quad (19)$$

In which, x_{sp} still denotes the setpoint, $\alpha \in [0, 1)$ is the soften factor, and the initial value of the trajectory is $w_{k|k} = x_k$. The value of α determines the speed of dynamic response and the curvature of the trajectory, the larger it is, the softer the curve is. Fig. 10. shows different trajectory with different α . Generally speaking, suitable α could be chosen based on the expected setting time in different practical cases.

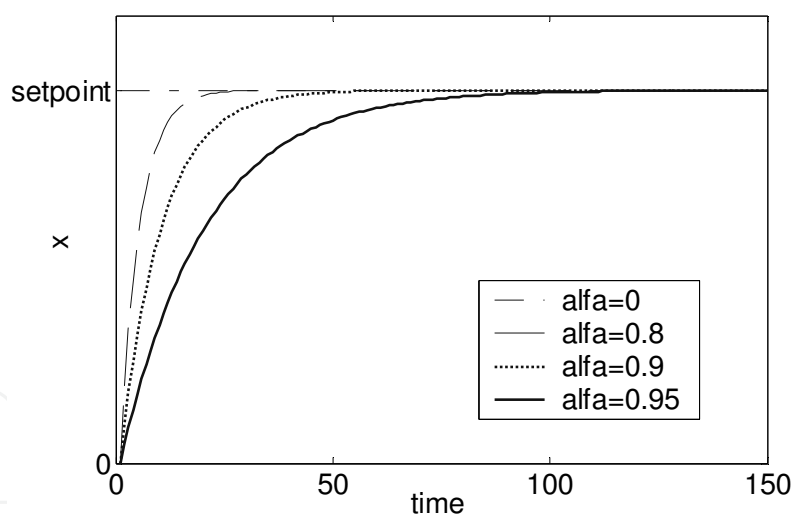


Fig. 10. Reference trajectory with different soften factor α

3.2 Stair-like control strategy

To lighten the computational load of nonlinear optimization, which is one of the biggest obstacles in NMPC's application, stair-like control strategy is introduced here. Suppose the first unknown control input's increment $\Delta u_k = u_k - u_{k-1} = \Delta$, and the stair coefficient β is a positive real number, then the future control input's increment can be decided by the following expression:

$$\Delta u_{k+j} = \beta \cdot \Delta u_{k+j-1} = \beta^j \cdot \Delta u_k = \beta^j \cdot \Delta, j = 1, 2, \dots, p - 1 \quad (20)$$

Instead of the full future sequence of control input's increment: $[\Delta u_k \ \Delta u_{k+1} \ \dots \ \Delta u_{k+p-1}]$, which has p independent variables. Using this strategy, in multi-step NMPC, it now need only compute Δu_k . The computational load now is independent of the length of predictive horizon, which is very convenient for us to choose long predictive horizon in NMPC to obtain a better control performance (Zheng *et al.*, 2007).

Since the dynamic optimization process will be repeated at every sample time, and only instant input $u_k = u_{k-1} + \Delta u_k$ will be carried out actually in NMPC, this strategy is efficient here. In the strategy, it supposes the future increase of control input will be in a same direction, which is the same as the experience in control practice of the human beings, and prevents the frequent oscillation of the input, which is very harmful to the actuators in real control plants. Fig. 11. shows the input sequences with different β .

3.3 Multi-step NMPC for affine system

The one-step NMPC in Section 2 is simple and fast, but it also has one fatal disadvantage. Its predictive horizon is only one step, while long predictive horizon is usually needed for better performance in MPC algorithms. One-step prediction may lead overshooting or other bad influence on system's behaviour. So we will try to establish a novel efficient multi-step NMPC based on proposed one-step NMPC in this section.

In this multi-step NMPC algorithm, the first step prediction is the same as (5), then follows the prediction of $\hat{x}_{k+1|k}$ in (5), the one-step prediction of $\hat{x}_{k+j|k}, j=2,3,\dots,p$ could be obtained directly:

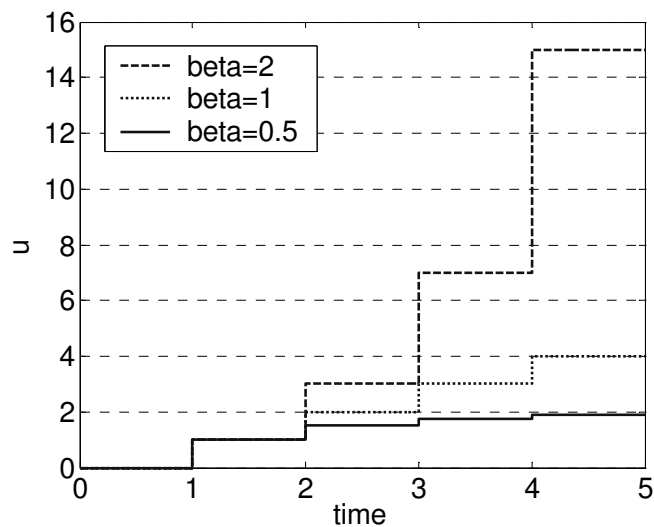


Fig. 11. Stair-like control strategy

$$\hat{x}_{k+j|k} = f(\hat{x}_{k+j-1|k}) + g(\hat{x}_{k+j-1|k}) \cdot u_{k+j-1|k} \tag{21}$$

Since $\hat{x}_{k+j-1|k}$ already contains nonlinear function of former data, one may not obtain the analytic solution of (21) for prediction more than one step. Take the situation of $j=2$ for example:

$$\hat{x}_{k+2|k} = f(\hat{x}_{k+1|k}) + g(\hat{x}_{k+1|k}) \cdot u_{k+1|k} = f(f(x_k) + g(x_k) \cdot u_{k|k}) + g(f(x_k) + g(x_k) \cdot u_{k|k}) \cdot u_{k+1|k} \tag{22}$$

For most nonlinear $f(\cdot)$ and $g(\cdot)$, the embedding form above makes it impossible to get an analytic solution of u_{k+1} and further future input. So, using reference trajectory, we modified the one-step predictions when $j \geq 2$ as follow:

$$\hat{x}_{k+j|k} = f(w_{k+j-1|k}) + g(w_{k+j-1|k}) \cdot u_{k+j-1} = f(w_{k+j-1|k}) + g(w_{k+j-1|k}) \cdot (u_{k-1} + \sum_{i=0}^{j-1} \Delta u_{k+i|k}) \tag{23}$$

Using the stair-like control strategy, mark $\Delta u_{k|k} = \Delta$, (23) can be transformed as:

$$\begin{aligned} \hat{x}_{k+j|k} &= f(w_{k+j-1|k}) + g(w_{k+j-1|k}) \cdot (u_{k-1} + \sum_{i=0}^{j-1} \beta^i \Delta) \\ &= f(w_{k+j-1|k}) + g(w_{k+j-1|k}) \cdot u_{k-1} + g(w_{k+j-1|k}) \cdot \sum_{i=0}^{j-1} \beta^i \Delta \\ &= \hat{x}_{k+j|k}^1 + g(w_{k+j-1|k}) \cdot \sum_{i=0}^{j-1} \beta^i \Delta \end{aligned} \tag{24}$$

Here, $\hat{x}_{k+j|k}^1$ contains only the known data at time k , while the other part is made up by the increment of future input, thus the unknown data are separated linearly by (24), so the analytic solution of Δ can be achieved.

For $j = 1, 2, \dots, p$, write the predictions in the form of matrix:

$$\hat{X}_k = \begin{bmatrix} \hat{x}_{k+1|k} \\ \hat{x}_{k+2|k} \\ \vdots \\ \hat{x}_{k+p|k} \end{bmatrix} \quad X_k^1 = \begin{bmatrix} \hat{x}_{k+1|k}^1 \\ \hat{x}_{k+2|k}^1 \\ \vdots \\ \hat{x}_{k+p|k}^1 \end{bmatrix} \quad W_k = \begin{bmatrix} w_{k+1|k} \\ w_{k+2|k} \\ \vdots \\ w_{k+p|k} \end{bmatrix} \quad \Delta U_k = \begin{bmatrix} \Delta u_{k|k} \\ \Delta u_{k+1|k} \\ \vdots \\ \Delta u_{k+p-1|k} \end{bmatrix} = \begin{bmatrix} \Delta \\ \beta \Delta \\ \vdots \\ \beta^{p-1} \Delta \end{bmatrix}$$

$$S_k = \begin{bmatrix} g(w_{k|k}) = g(x_k) & 0 & \dots & 0 \\ g(w_{k+1|k}) & g(w_{k+1|k}) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ g(w_{k+p-1|k}) & g(w_{k+p-1|k}) & \dots & g(w_{k+p-1|k}) \end{bmatrix} = \begin{bmatrix} s_1 & 0 & \dots & 0 \\ s_2 & s_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ s_p & s_p & \dots & s_p \end{bmatrix}$$

$$S_k \cdot \Delta U_k = \begin{bmatrix} s_1 \\ s_2(1+\beta) \\ \vdots \\ s_p(1+\beta+\dots+\beta^{p-1}) \end{bmatrix} \Delta = \bar{S}_k \cdot \Delta \quad (25)$$

Thus $\hat{X}_k = X_k^1 + S_k \cdot \Delta U_k = X_k^1 + \bar{S}_k \cdot \Delta$, for minimization of traditional quadric objective function $\min_{\Delta} J_k = \min_{\Delta} [(\hat{X}_k - W_k)^T Q (\hat{X}_k - W_k) + \Delta U_k^T R \Delta U_k]$, where semi-positive definite matrix Q and positive definite matrix R are weighting matrices, by $\frac{\partial J_k}{\partial \Delta} = 0$ and $\frac{\partial^2 J_k}{\partial \Delta^2} > 0$, the control solution of multi-step prediction is then obtained. Especially for single input problem, with objective function $\min_{\Delta} J_k = \min_{\Delta} [(\hat{X}_k - W_k)^T (\hat{X}_k - W_k) + r \Delta U_k^T \Delta U_k]$, it is easily denoted as follow:

$$\Delta = \frac{\bar{S}_k^T (W_k - X_k^1)}{\bar{S}_k^T \bar{S}_k + r(1 + \beta^2 + \dots + \beta^{2(p-1)})} \quad (26)$$

At last, the instant input $u_{k|k} = u_{k-1} + \Delta$ can be carried out actually. As mentioned in Section 2, and if the model mismatch can be seen as time-invariant in p sample time (usually satisfied in the case of steady state in practice), to maintain the robustness, e_k or \bar{e}_k can be also added to every prediction as mentioned in (17) and (18):

$$\begin{aligned} \hat{x}_{k+1|k} &= f(x_k) + g(x_k) \cdot u_{k|k} + \bar{e}_k \\ \hat{x}_{k+j|k} &= f(w_{k+j-1|k}) + g(w_{k+j-1|k}) \cdot u_{k+j-1|k} + \bar{e}_k, j = 2, 3, \dots, p \end{aligned} \quad (27)$$

Though there are approximate predictions in the novel NMPC which may take in some inaccuracy, the feedback compensation mentioned above and the new optimization process at every sample time will eliminate the error before its accumulation, to keep the efficiency of the algorithm. The constraints also could be handled by methods mentioned Section 2 or by other numerical optimizing algorithm, thus we would not discuss about it here again.

3.4 Multi-step NMPC of the water-tank system

Choose $\alpha = 0.975$, $\beta = 0.5$, $r = 0.005$, $x_{sp} = 60\%$ or 30% and predictive horizon $p = 10$ to carry out simulations. Still use the different plant model and predictive model as that of Fig. 5. and Fig. 6. to imitate the model mismatch, the result in Fig. 12. and Fig. 13. shows the efficiency and robustness of this efficient multi-objective NMPC.

Choose $\alpha = 0.975$, $\beta = 0.5$, $r = 0.005$, $x_{sp} = 60\%$ to carry out experiments. Comparing control result between one-step NMPC and multi-step NMPC in Fig. 14. and Fig. 15., we can see the obvious developments on both input and output of the water-tank system when longer predictive horizon is used. It also verifies the efficiency of proposed novel multi-step NMPC algorithm. At last, Fig. 16. is the satisfactory performance of the efficient multi-step NMPC under disturbance (we open an additional outlet valve of the tank for 20 seconds).

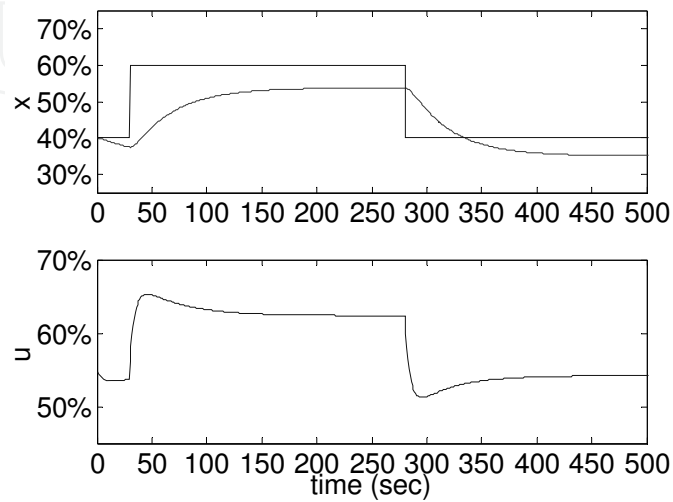


Fig. 12. Simulation of multi-step NMPC with model mismatch but without feedback compensation

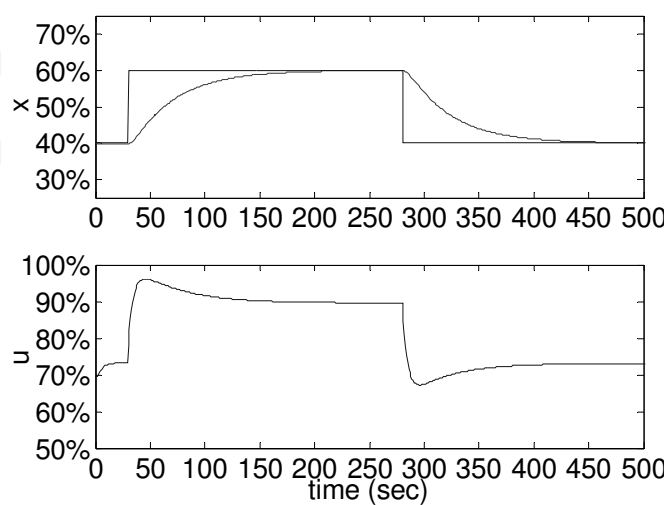


Fig. 13. Simulation of multi-step NMPC with model mismatch and feedback compensation

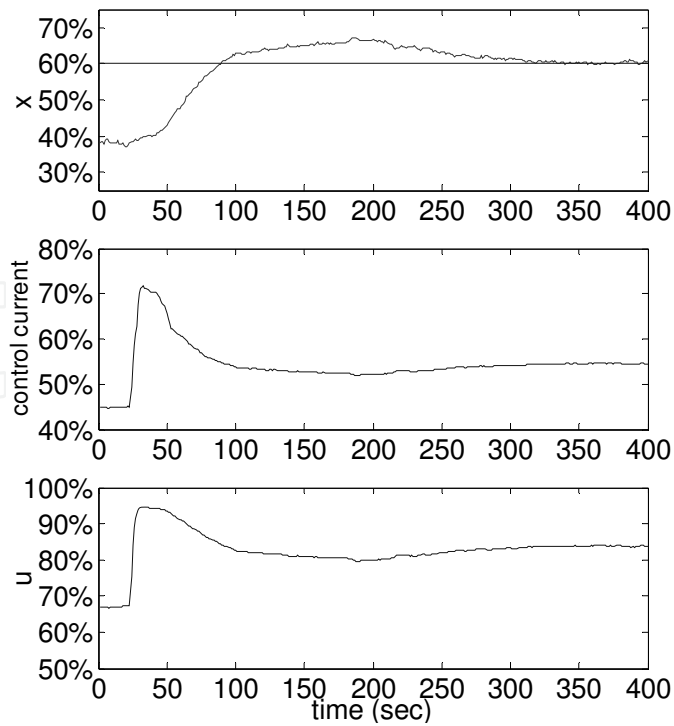


Fig. 14. Experiment of one-step NMPC with setpoint $x_{sp} = 60\%$ (Overshooting exists when setpoint is higher than Fig. 9.)

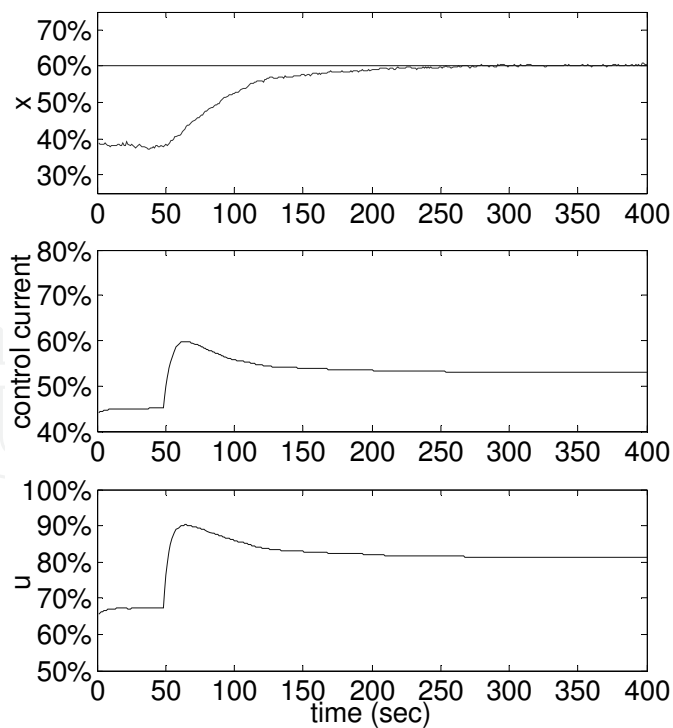


Fig. 15. Experiment of one-step NMPC with setpoint $x_{sp} = 60\%$ ($p=10$ and No overshooting)

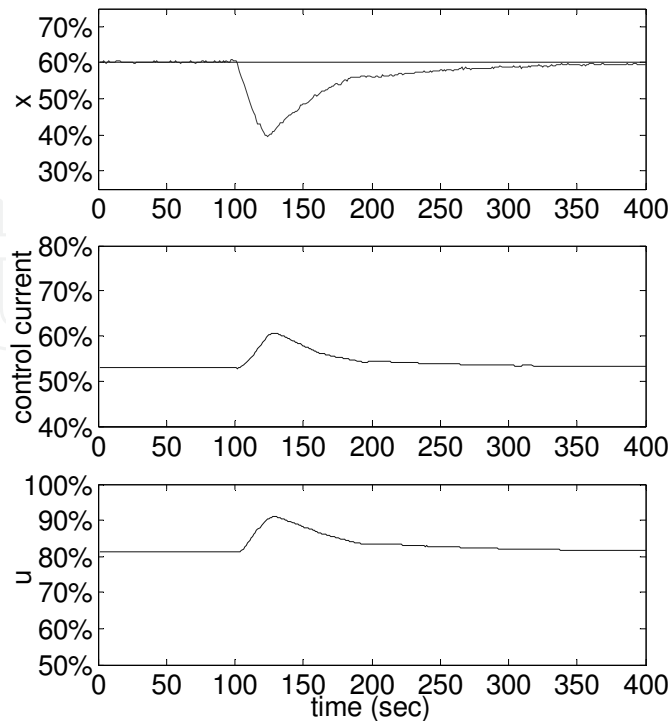


Fig. 16. Experiment of one-step NMPC under disturbance

4. Conclusion and acknowledgement

Using a series of approximate one-step predictions instead of the traditional multi-step prediction, the proposed multi-step NMPC led to an analytic result for nonlinear control of affine system. The use of stair-like control strategy caused a very little computational load and the feedback compensation brought robustness of model mismatch to it.

The simulations and experiments verify the practicability and efficiency of this multi-step NMPC for affine system, while the theoretical stability and other analysis will be the future work with considerable value.

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Model Predictive Control (MPC) refers to a class of control algorithms in which a dynamic process model is used to predict and optimize process performance. From lower request of modeling accuracy and robustness to complicated process plants, MPC has been widely accepted in many practical fields. As the guide for researchers and engineers all over the world concerned with the latest developments of MPC, the purpose of "Advanced Model Predictive Control" is to show the readers the recent achievements in this area. The first part of this exciting book will help you comprehend the frontiers in theoretical research of MPC, such as Fast MPC, Nonlinear MPC, Distributed MPC, Multi-Dimensional MPC and Fuzzy-Neural MPC. In the second part, several excellent applications of MPC in modern industry are proposed and efficient commercial software for MPC is introduced. Because of its special industrial origin, we believe that MPC will remain energetic in the future.

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