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Distributed Nonlinear Filtering Under Packet Drops and Variable Delays for Robotic Visual Servoing

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1. Introduction

State estimation over communication networks is in use by many robotic applications in industry, in defense systems, as well as in several exploration and surveillance tasks. The incorporation of a communication network in the control loop has enabled to perform multi-sensor fusion and distributed information processing, thus improving significantly the autonomy and reliability of robotic systems (Medeiros et al., 2008), (Olfati-Saber, 2006), (Watanabe & Tzafestas, 1992). It has been shown that scalable distributed state estimation can be achieved for robotic models, when the measurements are linear functions of the state and the associated process and measurement noise models follow a Gaussian distribution (Mahler, 2007), (Nettleton et al., 2003). The results have been also extended to the case of nonlinear non-Gaussian dynamical systems (Rigatos, 2010a), (Makarenko & Durrant-Whyte, 2006).
An issue which is associated to the implementation of such networked control systems is how to compensate for random delays and packet losses so as to enhance the accuracy of estimation and consequently to improve the stability of the control loop. The idea of incorporating delayed measurements within a Kalman Filter framework is a possible solution for the compensation of network-induced delays and packet losses, and is also known as update with out-of-sequence measurements (Bar Shalom, 2002). The solution proposed in (Bar Shalom, 2002) is optimal under the assumption that the delayed measurement was processed within the last sampling interval (one-step-lag problem). There have been also some attempts to extend these results to nonlinear state estimation (Golapalakrishnan et al., 2011), (Jia et al., 2008). More recently there has been research effort in the redesign of distributed Kalman Filtering algorithms for linear systems so as to eliminate the effects of delays in measurement transmissions and packet drops, while also alleviating the one-step-lag assumption (Xia et al., 2009). This chapter presents an approach to distributed state estimation-based control of nonlinear systems, capable of incorporating delayed measurements in the estimation algorithm while being also robust to packet losses.
First, the chapter examines the problem of distributed nonlinear filtering over a communication/sensors network, and the use of the estimated state vector in a control loop. As a possible filtering approach, the Extended Information Filter is proposed (Rigatos, 2010a). In the Extended Information Filter the local filters do not exchange raw measurements but send to an aggregation filter their local information matrices (local inverse covariance matrices which can be also associated to Fisher Information Matrices) and their associated

local information state vectors (products of the local information matrices with the local state vectors) (Lee, 2008). The Extended Information Filter performs fusion of state estimates from local distributed Extended Kalman Filters which in turn are based on the assumption of linearization of the system dynamics by first order Taylor series expansion and truncation of the higher order linearization terms. Moreover, the Extended Kalman Filter requires the computation of Jacobians which in the case of high order nonlinear dynamical systems can be a cumbersome procedure. This approach introduces cumulative errors to the state estimation performed by the local Extended Kalman Filter recursion which is finally transferred to the master filter where the aggregate state estimate of the controlled system is computed. Consequently, these local estimation errors may result in the deterioration of the performance of the associated control loop or even risk its stability (Rigatos, 2009),(Rigatos et al., 2009).

To overcome the aforementioned weaknesses of the Extended Information Filter a derivative-free approach to Extended Information Filtering has been proposed (Rigatos & Siano, 2010), (Rigatos, 2010c). The system is first subject to a linearization transformation and next state estimation is performed by applying the standard Kalman Filter to the linearized model. At a second level, the standard Information Filter is used to fuse the state estimates obtained from local derivative-free Kalman filters running at the local information processing nodes. This approach has significant advantages because unlike the Extended Information Filter (i) is not based on local linearization of the system dynamics (ii) it does not assume truncation of higher order Taylor expansion terms thus preserving the accuracy and robustness of the performed estimation, (iii) it does not require the computation of Jacobian matrices.

At a second stage the chapter proposes a method for the compensation of random delays and packet drops which may appear during the transmission of measurements and state vector estimates, and which the can cause the deterioration of the performance of the distributed filtering-based control scheme (Xia et al., 2009), (Schenato, 2007), (Schenato, 2008). Two cases are distinguished: (i) there are time delays and packet drops in the transmission of information between the distributed local filters and the master filter, (ii) there are time delays and packet drops in the transmission of information from distributed sensors to each one of the local filters. In the first case, the structure and calculations of the master filter for estimating the aggregate state vector remain unchanged. In the second case, the effect of the random delays and packets drops has to be taken into account in the redesign of the local Kalman Filters, which implies a modified Riccati equation for the computation of the covariance matrix of the state vector estimation error, as well as the use of a correction (smoothing) term in the update of the state vector's estimate so as to compensate for delayed measurements arriving at the local Kalman Filters.

Finally, the chapter shows that the aggregate state vector produced by a derivative-free Extended Information Filter, suitably modified to compensate for communication delays and packet drops, can be used for sensorless control and robotic visual servoing. The problem of visual servoing over a network of synchronised cameras has been previously studied in (Schuurman & Capson, 2004). In this chapter, visual servoing over a cameras network is considered for the nonlinear dynamic model of a planar single-link robotic manipulator. It is assumed that the network on which the visual servoing loop relies, can be affected by disturbances, such as random delays or loss of frames during their transmission to the local processing vision nodes. The position of the robot's end effector in the cartesian space (and equivalently the angle of the robotic link) is measured through m cameras. In turn, these measurements are processed by m distributed derivative-free Kalman Filters thus providing

m different estimates of the robotic link's state vector. Next, the local state estimates are fused with the use of the standard Information Filter. After all, the aggregate estimation of the state vector is used in a control loop which enables the robotic link to perform trajectory tracking.
The structure of the chapter is as follows: In Section 2 the Extended Kalman Filter is introduced and its use for state estimation of nonlinear dynamical systems is explained. In Section 3 a derivative-free Kalman Filtering approach to state estimation of nonlinear systems is analyzed. In Section 4 the derivative-free Extended Information Filter is formulated as an approach to distributed state estimation for nonlinear systems, capable of overcoming the drawbacks of the standard Extended Information Filter. In Section 5 the problem of distributed filtering under random delays and packet drops is analyzed. The results are also applied to distributed state estimation with the use of the derivative-free Extended Information Filter. In Section 6 the previously described approach for derivative-free Extended Information Filtering under communication delays and packet drops is applied to the problem of state estimation-based control of nonlinear systems. As a case study the model of a planar robot is considered, while the estimation of its state vector is performed with the use of distributed filtering through the processing of measurements provided by vision sensors (cameras). In Section 7 simulation tests are presented, to confirm the efficiency of the proposed derivative-free Extended Information Filtering method. Finally, in Section 8 concluding remarks are given.

2. Extended Kalman Filtering for nonlinear dynamical systems

2.1 The continuous-time Kalman Filter for the linear state estimation model

First, the continuous-time dynamical system of Eq. (1) is assumed (Rigatos & Tzafestas, 2007), (Rigatos, 2010d):

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + w(t), \ t \geq t_0 \\ z(t) = Cx(t) + v(t), \ t \geq t_0 \end{cases} \tag{1}$$

where $x \in R^{m \times 1}$ is the system's state vector, and $z \in R^{p \times 1}$ is the system's output. Matrices A,B and C can be time-varying and $w(t)$,$v(t)$ are uncorrelated white Gaussian noises. The covariance matrix of the process noise $w(t)$ is $Q(t)$, while the covariance matrix of the measurement noise is $R(t)$. Then, the Kalman Filter is a linear state observer which is given by

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + K[z - C\hat{x}], \ \hat{x}(t_0) = 0 \\ K(t) = PC^T R^{-1} \\ \dot{P} = AP + PA^T + Q - PC^T R^{-1} CP \end{cases} \tag{2}$$

where $\hat{x}(t)$ is the optimal estimation of the state vector $x(t)$ and $P(t)$ is the covariance matrix of the state vector estimation error with $P(t_0) = P_0$. The Kalman Filter consists of the system's state equation plus a corrective term $K[z - C\hat{x}]$. The selection of gain K corresponds actually to the solution of an optimization problem. This is expressed as the minimization of a quadratic cost functional and is performed through the solution of a Riccati equation. In that case the observer's gain K is calculated by $K = PC^T R^{-1}$ considering an optimal control problem for the dual system (A^T, C^T), where the covariance matrix of the estimation error P is found by the solution of a continuous-time Riccati equation of the form

$$\dot{P} = AP + PA^T + Q - PC^TR^{-1}CP \tag{3}$$

where matrices Q and R stand for the process and measurement noise covariance matrices, respectively.

2.2 The discrete-time Kalman Filter for linear dynamical systems

In the discrete-time case a dynamical system is assumed to be expressed in the form of a discrete-time state model (Rigatos & Tzafestas, 2007), (Rigatos, 2010d):

$$\begin{aligned} x(k+1) &= A(k)x(k) + L(k)u(k) + w(k) \\ z(k) &= Cx(k) + v(k) \end{aligned} \tag{4}$$

where the state $x(k)$ is a m-vector, $w(k)$ is a m-element process noise vector and A is a $m \times m$ real matrix. Moreover the output measurement $z(k)$ is a p-vector, C is an $p \times m$-matrix of real numbers, and $v(k)$ is the measurement noise. It is assumed that the process noise $w(k)$ and the measurement noise $v(k)$ are uncorrelated.

Now the problem of interest is to estimate the state $x(k)$ based on the sequence of output measurements $z(1), z(2), \cdots, z(k)$. The initial value of the state vector $x(0)$, and the initial value of the error covariance matrix $P(0)$ is unknown and an estimation of it is considered, i.e. $\hat{x}(0)$= a guess of $E[x(0)]$ and $\hat{P}(0)$= a guess of $Cov[x(0)]$.

For the initialization of matrix P one can set $\hat{P}(0) = \lambda I$, with $\lambda > 0$. The state vector $x(k)$ has to be estimated taking into account $\hat{x}(0)$, $\hat{P}(0)$ and the output measurements $Z = [z(1), z(2), \cdots, z(k)]^T$, i.e. $\hat{x}(k) = \alpha_n(\hat{x}(0)), \hat{P}(0), Z(k))$. This is a linear minimum mean squares estimation problem (LMMSE) formulated as $\hat{x}(k+1) = a_{n+1}(\hat{x}(k), z(k+1))$. The process and output noise are white and their covariance matrices are given by: $E[w(i)w^T(j)] = Q\delta(i-j)$ and $E[v(i)v^T(j)] = R\delta(i-j)$.

Using the above, the discrete-time Kalman filter can be decomposed into two parts: i) time update (prediction stage), and ii) measurement update (correction stage). The first part employs an estimate of the state vector $x(k)$ made before the output measurement $z(k)$ is available (a priori estimate). The second part estimates $x(k)$ after $z(k)$ has become available (a posteriori estimate).

- When the set of measurements $Z^- = \{z(1), \cdots, z(k-1)\}$ is available. From Z^- an *a priori* estimation of $x(k)$ is obtained which is denoted by $\hat{x}^-(k)$= the estimate of $x(k)$ given Z^-.
- When $z(k)$ is available, the output measurements set becomes $Z = \{z(1), \cdots, z(k)\}$, where $\hat{x}(k)$= the estimate of $x(k)$ given Z.

The associated estimation errors are defined by $e^-(k) = x(k) - \hat{x}^-(k)$= the a priori error, and $e(k) = x(k) - \hat{x}(k)$= the a posteriori error. The estimation error covariance matrices associated with $\hat{x}(k)$ and $\hat{x}(k)$ are defined as $P^-(k) = Cov[e^-(k)] = E[e^-(k)e^-(k)^T]$ and $P(k) = Cov[e(k)] = E[e(k)e^T(k)]$ (Kamen & Su, 1999). From the definition of the trace of a matrix, the mean square error of the estimates can be written as $MSE(\hat{x}^-(k)) = E[e^-(k)e^-(k)^T] = tr(P^-(k))$ and $MSE(x(k)) = E[e(k)e^T(k) = tr(P(k))$.

Finally, the linear Kalman filter equations in cartesian coordinates are *measurement update*:

$$\begin{aligned} K(k) &= P^-(k)C^T[C\cdot P^-(k)C^T + R]^{-1} \\ \hat{x}(k) &= \hat{x}^-(k) + K(k)[z(k) - C\hat{x}^-(k)] \\ P(k) &= P^-(k) - K(k)CP^-(k) \end{aligned} \tag{5}$$

time update:

$$\begin{aligned} P^-(k+1) &= A(k)P(k)A^T(k) + Q(k) \\ \hat{x}^-(k+1) &= A(k)\hat{x}(k) + L(k)u(k) \end{aligned} \tag{6}$$

2.3 The extended Kalman Filter

State estimation can be also performed for nonlinear dynamical systems using the Extended Kalman Filter recursion (Ahrens & Khalil, 2005), (Boutayeb et al., 1997). The following nonlinear state model is considered (Rigatos, 2010a), (Rigatos & Tzafestas, 2007):

$$\begin{aligned} x(k+1) &= \phi(x(k)) + L(k)u(k) + w(k) \\ z(k) &= \gamma(x(k)) + v(k) \end{aligned} \tag{7}$$

where $x \in R^{m\times 1}$ is the system's state vector and $z \in R^{p\times 1}$ is the system's output, while $w(k)$ and $v(k)$ are uncorrelated, zero-mean, Gaussian zero-mean noise processes with covariance matrices $Q(k)$ and $R(k)$ respectively. The operators $\phi(x)$ and $\gamma(x)$ are vectors defined as $\phi(x) = [\phi_1(x), \phi_2(x), \cdots, \phi_m(x)]^T$, and $\gamma(x) = [\gamma_1(x), \gamma_2(x), \cdots, \gamma_p(x)]^T$, respectively. It is assumed that ϕ and γ are sufficiently smooth in x so that each one has a valid series Taylor expansion. Following a linearization procedure, ϕ is expanded into Taylor series about $\hat{x}$:

$$\phi(x(k)) = \phi(\hat{x}(k)) + J_\phi(\hat{x}(k))[x(k) - \hat{x}(k)] + \cdots \tag{8}$$

where $J_\phi(x)$ is the Jacobian of ϕ calculated at $\hat{x}(k)$:

$$J_\phi(x) = \frac{\partial \phi}{\partial x}|_{x=\hat{x}(k)} = \begin{pmatrix} \frac{\partial \phi_1}{\partial x_1} & \frac{\partial \phi_1}{\partial x_2} & \cdots & \frac{\partial \phi_1}{\partial x_m} \\ \frac{\partial \phi_2}{\partial x_1} & \frac{\partial \phi_2}{\partial x_2} & \cdots & \frac{\partial \phi_2}{\partial x_m} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \phi_m}{\partial x_1} & \frac{\partial \phi_m}{\partial x_2} & \cdots & \frac{\partial \phi_m}{\partial x_m} \end{pmatrix} \tag{9}$$

Likewise, γ is expanded about $\hat{x}^-(k)$

$$\gamma(x(k)) = \gamma(\hat{x}^-(k)) + J_\gamma[x(k) - \hat{x}^-(k)] + \cdots \tag{10}$$

where $\hat{x}^-(k)$ is the estimation of the state vector $x(k)$ before measurement at the k-th instant to be received and $\hat{x}(k)$ is the updated estimation of the state vector after measurement at the k-th instant has been received. The Jacobian $J_\gamma(x)$ is

$$J_\gamma(x) = \frac{\partial \gamma}{\partial x}|_{x=\hat{x}^-(k)} = \begin{pmatrix} \frac{\partial \gamma_1}{\partial x_1} & \frac{\partial \gamma_1}{\partial x_2} & \cdots & \frac{\partial \gamma_1}{\partial x_m} \\ \frac{\partial \gamma_2}{\partial x_1} & \frac{\partial \gamma_2}{\partial x_2} & \cdots & \frac{\partial \gamma_2}{\partial x_m} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \gamma_p}{\partial x_1} & \frac{\partial \gamma_p}{\partial x_2} & \cdots & \frac{\partial \gamma_p}{\partial x_m} \end{pmatrix} \tag{11}$$

The resulting expressions create first order approximations of ϕ and γ. Thus the linearized version of the system is obtained:

$$\begin{array}{l} x(k+1) = \phi(\hat{x}(k)) + J_{\phi}(\hat{x}(k))[x(k) - \hat{x}(k)] + w(k) \\ z(k) = \gamma(\hat{x}^{-}(k)) + J_{\gamma}(\hat{x}^{-}(k))[x(k) - \hat{x}^{-}(k)] + v(k) \end{array} \quad (12)$$

Now, the EKF recursion is as follows: First the time update is considered: by $\hat{x}(k)$ the estimation of the state vector at instant k is denoted. Given initial conditions $\hat{x}^{-}(0)$ and $P^{-}(0)$ the recursion proceeds as:

- *Measurement update*. Acquire $z(k)$ and compute:

$$\begin{array}{l} K(k) = P^{-}(k)J_{\gamma}^{T}(\hat{x}^{-}(k)) \cdot [J_{\gamma}(\hat{x}^{-}(k))P^{-}(k)J_{\gamma}^{T}(\hat{x}^{-}(k)) + R(k)]^{-1} \\ \hat{x}(k) = \hat{x}^{-}(k) + K(k)[z(k) - \gamma(\hat{x}^{-}(k))] \\ P(k) = P^{-}(k) - K(k)J_{\gamma}(\hat{x}^{-}(k))P^{-}(k) \end{array} \quad (13)$$

- *Time update*. Compute:

$$\begin{array}{l} P^{-}(k+1) = J_{\phi}(\hat{x}(k))P(k)J_{\phi}^{T}(\hat{x}(k)) + Q(k) \\ \hat{x}^{-}(k+1) = \phi(\hat{x}(k)) + L(k)u(k) \end{array} \quad (14)$$

The schematic diagram of the EKF loop is given in Fig. 1.

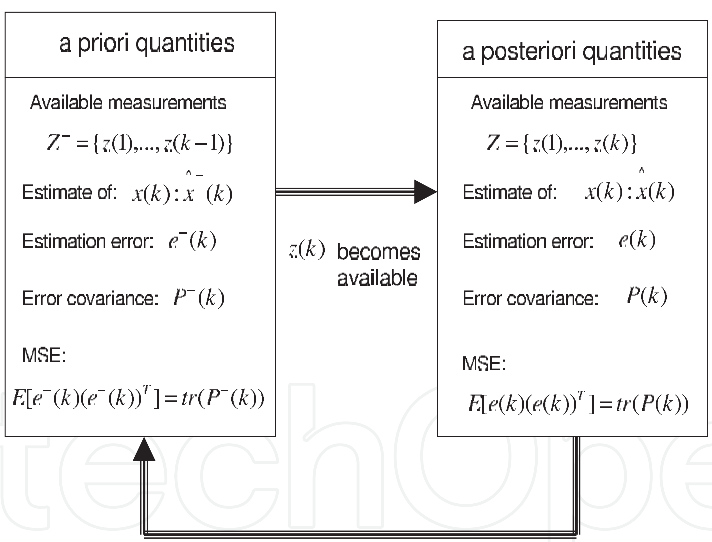


Fig. 1. Schematic diagram of the EKF loop

3. Derivative-free Kalman Filtering for a class of nonlinear systems

3.1 State estimator design through a nonlinear transformation

It will be shown that through a nonlinear transformation it is possible to design a state estimator for a class of nonlinear systems, which can substitute for the Extended Kalman Filter. The results will be generalized towards derivative-free Kalman Filtering for nonlinear systems. The following continuous-time nonlinear single-output system is considered (Marino, 1990),(Marino & Tomei, 1992)

$$\begin{array}{l}\dot{x} = f(x) + q_0(x,u) + \sum_{i=1}^{p}\theta_i q_i(x,u), \text{ or} \\ \dot{x} = f(x) + q_0(x,u) + Q(x,u)\theta \; x\in R^n,\; u\in R^m,\; \theta\in R^p \\ z = h(x),\; z\in R\end{array} \tag{15}$$

with $q_i: R^n\times R^m \to R^n$, $0\leq i\leq p$, $f: R^n\to R^n$, $h: R^n\to R$, smooth functions, $h(x_0) = 0$, $q_0(x,0) = 0$ for every $x \in R^n$; x is the state vector, $u(x,t): R^+\to R^m$ is the control which is assumed to be known, θ is the parameter vector which is supposed to be constant and y is the scalar output.
The first main assumption on the class of systems considered is the linear dependence on the parameter vector θ. The second main assumption requires that systems of Eq.(15) are transformable by a parameter independent state-space change of coordinates in R^n

$$\zeta = T(x),\;\; T(x_0) = 0 \tag{16}$$

into the system

$$\begin{array}{l}\dot{\zeta} = A_c\zeta + \psi_0(z,u) + \sum_{i=1}^{p}\theta_i\psi_i(z,u)\Rightarrow \\ \dot{\zeta} = A_c\zeta + \psi_0(z,u) + \Psi(z,u)\theta \\ \\ z = C_c\zeta\end{array} \tag{17}$$

with

$$A_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \tag{18}$$

$$C_c = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \end{pmatrix} \tag{19}$$

and $\psi_i: R\times R^m\to R^n$ smooth functions for $i = 0,\cdots,p$. The necessary and sufficient conditions for the initial nonlinear system to be transformable into the form of Eq.(17) have been given in (Marino, 1990),(Marino & Tomei, 1992), and are summarized in the following:

(i) rank$\{dh(x), d_{L_f}h(x), \cdots, d_{L_f^{n-1}}h(x)\} = n$, $\forall x\in R^n$ (which implies local observability). It is noted that $L_f h(x)$ stands for the Lie derivative $L_f h(x) = (\nabla h)f$ and the repeated Lie derivatives are recursively defined as $L_f^0 h = h$ for $i = 0$, $L_f^i h = L_f L_f^{i-1}h = \nabla L_f^{i-1}hf$ for $i = 1,2,\cdots$.
(ii) $[ad_f^i g, ad_f^j g] = 0$, $0\leq i,\; j\leq n-1$. It is noted that $ad_f^i g$ stands for a Lie Bracket which is defined recursively as $ad_f^i g = [f, ad_f^{i-1}]g$ with $ad_f^0 g = g$ and $ad_f g = [f,g] = \nabla g f - \nabla f g$.
(iii) $[q_i, ad_f^i g] = 0$, $0\leq i\leq p$, $0\leq j\leq n-2\;\forall\; u\in R^m$.
(iv) the vector fields $ad_f^i g$, $0\leq i\leq n-1$ are complete, in which g is the vector field satisfying

$$< \begin{pmatrix} dh \\ \vdots \\ d(L_f^{n-1}h) \end{pmatrix}, g >= \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix} \tag{20}$$

Then for every parameter vector θ, the system

$$\begin{array}{l} \dot{\hat{\zeta}} = A_c\hat{\zeta} + \psi_0(z,u) + \sum_{i=1}^{p}\theta_i\psi_i(z,u) + K(z - C_c\hat{\zeta}) \\ \hat{x} = T^{-1}(\hat{\zeta}) \end{array} \tag{21}$$

is an asymptotic observer for a suitable choice of K provided that the state $x(t)$ is bounded, with estimation error dynamics

$$\dot{e} = (A_c - KC_c)e = \begin{pmatrix} -k_1 & 1 & 0 & \cdots & 0 \\ -k_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -k_{n-1} & 0 & 0 & \cdots & 1 \\ -k_n & 0 & 0 & \cdots & 0 \end{pmatrix} e \tag{22}$$

The eigenvalues of $A_c - KC_c$ can be arbitrarily placed by choosing the vector K, since they coincide with the roots of the polynomial $s^n + k_1 s^{n-1} + \cdots + k_n$.

3.2 Derivative-free Kalman Filtering for nonlinear systems

Since Eq. (21) provides an asympotic observer for the initial nonlinear system of Eq. (15) one can consider a case in which the observation error gain matrix K can be provided by the Kalman Filter equations given initially in the continuous-time KF formulation, or in discrete-time form by Eq. (5) and Eq. (6). The following single-input single-output nonlinear dynamical system is considered

$$x^{(n)} = f(x,t) + g(x,t)u(x,t) \tag{23}$$

where $z = x$ is the system's output, and $f(x,t)$, $g(x,t)$ are nonlinear functions. It can be noticed that the system of Eq. (23) belongs to the general class of systems of Eq. (15). Assuming the transformation $\zeta_i = x^{(i-1)}$, $i = 1, \cdots, n$, and $x^{(n)} = f(x,t) + g(x,t)u(x,t) = v(\zeta,t)$, i.e. $\dot{\zeta}_n = v(\zeta,t)$, one obtains the linearized system of the form

$$\begin{array}{l} \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = \zeta_3 \\ \cdots\ \cdots \\ \dot{\zeta}_{n-1} = \zeta_n \\ \dot{\zeta}_n = v(\zeta,t) \end{array} \tag{24}$$

which in turn can be written in state-space equations as

$$\begin{pmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ \cdots \\ \dot{\zeta}_{n-1} \\ \dot{\zeta}_n \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \cdots \\ \zeta_{n-1} \\ \zeta_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{pmatrix} v(\zeta,t) \tag{25}$$

$$z = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \end{pmatrix} \zeta \tag{26}$$

The system of Eq. (25) and Eq. (26) has been written in the form of Eq. (17), which means that Eq. (21) is the associated asymptotic observer. Therefore, the observation gain K appearing in Eq. (21) can be found using either linear observer design methods (in that case the elements of the observation error gain matrix K have fixed values), or the recursive calculation of the continuous-time Kalman Filter gain described in subsection 2.2. If the discrete-time Kalman Filter is to be used then one has to apply the recursive formulas of Eq. (5) and Eq. (6) on the discrete-time equivalent of Eq. (25) and Eq. (26).

4. Derivative-free Extended Information Filter

4.1 Calculation of local estimations in terms of EIF information contributions

Again the discrete-time nonlinear system of Eq. (7) is considered. The Extended Information Filter (EIF) performs fusion of local state vector estimates which are provided by local Extended Kalman Filters, using the *Information matrix* and the *Information state vector* (Lee, 2008), (Manyika & Durrant-Whyte, 1994). The Information Matrix is the inverse of the state vector covariance matrix, and can be also associated to the Fisher Information matrix (Rigatos & Zhang, 2009). The Information state vector is the product between the Information matrix and the local state vector estimate

$$\begin{array}{c} Y(k) = P^{-1}(k) = I(k) \\ \hat{y}(k) = P^{-}(k)^{-1}\hat{x}(k) = Y(k)\hat{x}(k) \end{array} \tag{27}$$

The update equation for the Information Matrix and the Information state vector are given by

$$\begin{aligned} Y(k) &= P^{-}(k)^{-1} + J_{\gamma}^{T}(k)R^{-1}(k)J_{\gamma}(k) \\ &= Y^{-}(k) + I(k) \end{aligned} \tag{28}$$

$$\begin{aligned} \hat{y}(k) &= \hat{y}^{-}(k) + J_{\gamma}^{T}(k)R(k)^{-1}[z(k) - \gamma(x(k)) + J_{\gamma}(k)\hat{x}^{-}(k)] \\ &= \hat{y}^{-}(k) + i(k) \end{aligned} \tag{29}$$

where

$$\begin{array}{c} I(k) = J_{\gamma}^{T}(k)R(k)^{-1}J_{\gamma}(k) \text{ is the associated information matrix and} \\ i(k) = J_{\gamma}^{T}(k)R(k)^{-1}[(z(k) - \gamma(x(k))) + J_{\gamma}\hat{x}^{-}(k)] \text{ is the information state contribution} \end{array} \tag{30}$$

The predicted information state vector and Information matrix are obtained from

$$\begin{array}{c} \hat{y}^{-}(k) = P^{-}(k)^{-1}\hat{x}^{-}(k) \\ Y^{-}(k) = P^{-}(k)^{-1} = [J_{\phi}(k)P^{-}(k)J_{\phi}(k)^{T} + Q(k)]^{-1} \end{array} \tag{31}$$

The Extended Information Filter is next formulated for the case that multiple local sensor measurements and local estimates are used to increase the accuracy and reliability of the estimation. It is assumed that an observation vector $z^{i}(k)$ is available for N different sensor sites $i = 1, 2, \cdots, N$ and each sensor observes a common state according to the local observation model, expressed by

$$z^{i}(k) = \gamma(x(k)) + v^{i}(k),\ i = 1, 2, \cdots, N \tag{32}$$

where the local noise vector $v^{i}(k) \sim N(0, R^{i})$ is assumed to be white Gaussian and uncorrelated between sensors. The variance of a composite observation noise vector v_k is expressed in terms of the block diagonal matrix

$$R(k) = diag[R^{1}(k), \cdots, R^{N}(k)]^{T} \tag{33}$$

The information contribution can be expressed by a linear combination of each local information state contribution i^i and the associated information matrix I^i at the i-th sensor site

$$\begin{array}{c} i(k) = \sum_{i=1}^{N} {J_\gamma^i}^T(k) R^i(k)^{-1}[z^i(k) - \gamma^k(x(k)) + J_\gamma^i(k)\hat{x}^-(k)] \\ I(k) = \sum_{i=1}^{N} {J_\gamma^i}^T(k) R^i(k)^{-1} J_\gamma^i(k) \end{array} \quad (34)$$

Using Eq. (34) the update equations for fusing the local state estimates become

$$\begin{array}{c} \hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^{N} {J_\gamma^i}^T(k) R^i(k)^{-1}[z^i(k) - \gamma^k(x(k)) + J_\gamma^i(k)\hat{x}^-(k)] \\ Y(k) = Y^-(k) + \sum_{i=1}^{N} {J_\gamma^i}^T(k) R^i(k)^{-1} J_\gamma^i(k) \end{array} \quad (35)$$

It is noted that in the Extended Information Filter an aggregation (master) fusion filter produces a global estimate by using the local sensor information provided by each local filter.

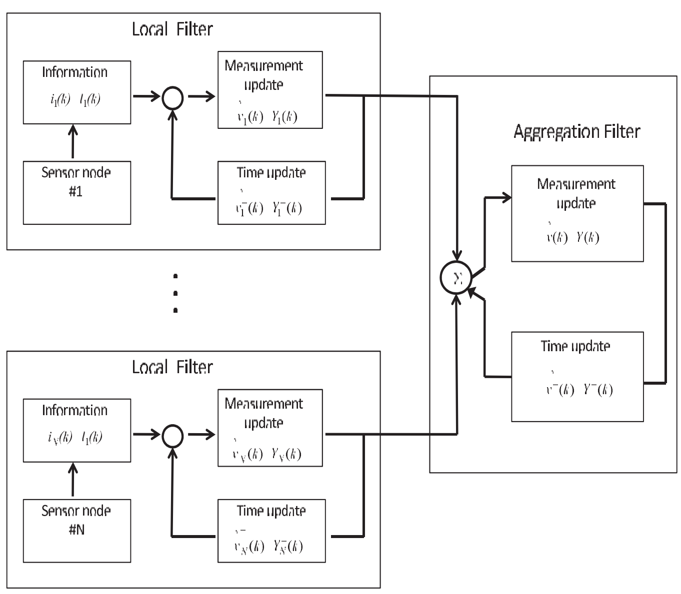


Fig. 2. Fusion of the distributed state estimates with the use of the Extended Information Filter

As in the case of the Extended Kalman Filter the local filters which constitute the Extended Information Filter can be written in terms of *time update* and a *measurement update* equation.

Measurement update: Acquire $z(k)$ and compute

$$\begin{array}{c} Y(k) = P^-(k)^{-1} + J_\gamma^T(k) R(k)^{-1} J_\gamma(k) \\ \text{or } Y(k) = Y^-(k) + I(k) \text{ where } I(k) = J_\gamma^T(k) R^{-1}(k) J_\gamma(k) \end{array} \quad (36)$$

$$\begin{array}{c} \hat{y}(k) = \hat{y}^-(k) + J_\gamma^T(k) R(k)^{-1}[z(k) - \gamma(\hat{x}(k)) + J_\gamma(k)\hat{x}^-(k)] \\ \text{or } \hat{y}(k) = \hat{y}^-(k) + i(k) \end{array} \quad (37)$$

Time update: Compute

$$Y^-(k+1) = P^-(k+1)^{-1} = [J_\phi(k) P(k) J_\phi(k)^T + Q(k)]^{-1} \quad (38)$$

$$y^-(k+1) = P^-(k+1)^{-1}\hat{x}^-(k+1) \tag{39}$$

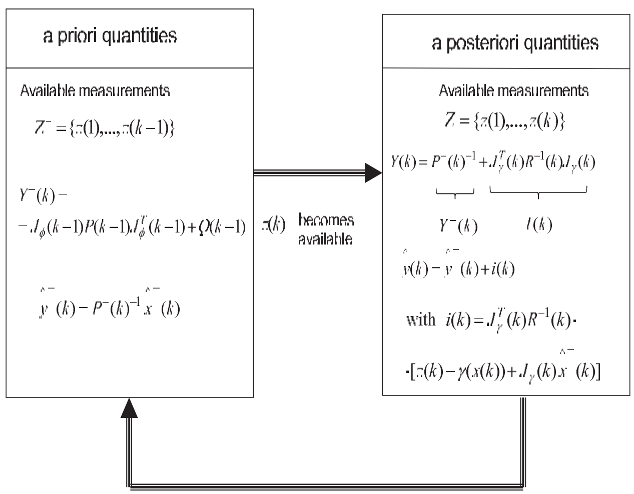


Fig. 3. Schematic diagram of the Extended Information Filter loop

4.2 Extended Information Filtering for state estimates fusion

In the Extended Information Filter each one of the local filters operates independently, processing its own local measurements. It is assumed that there is no sharing of measurements between the local filters and that the aggregation filter (Fig. 2) does not have direct access to the raw measurements giving input to each local filter. The outputs of the local filters are treated as measurements which are forwarded to the aggregation fusion filter (Lee, 2008). Then each local filter is expressed by its respective error covariance and estimate in terms of information contributions given in Eq.(48)

$$\begin{aligned} P_i^{-1}(k) &= P_i^-(k)^{-1} + J_\gamma^T(k)R(k)^{-1}J_\gamma(k) \\ \hat{x}_i(k) &= P_i(k)(P_i^-(k)^{-1}\hat{x}_i^-(k)) + J_\gamma^T(k)R(k)^{-1}[z^i(k) - \gamma^k(x(k)) + J_\gamma^i(k)\hat{x}_i^-(k)] \end{aligned} \tag{40}$$

It is noted that the local estimates are suboptimal and also conditionally independent given their own measurements. The global estimate and the associated error covariance for the aggregate fusion filter can be rewritten in terms of the computed estimates and covariances from the local filters using the relations

$$\begin{aligned} J_\gamma^T(k)R(k)^{-1}J_\gamma(k) &= P_i(k)^{-1} - P_i^-(k)^{-1} \\ J_\gamma^T(k)R(k)^{-1}[z^i(k) - \gamma^k(x(k)) + J_\gamma^i(k)\hat{x}^-(k)] &= P_i(k)^{-1}\hat{x}_i(k) - P_i(k)^{-1}\hat{x}_i(k-1) \end{aligned} \tag{41}$$

For the general case of N local filters $i = 1, \cdots, N$, the distributed filtering architecture is described by the following equations

$$\begin{aligned} P(k)^{-1} &= P^-(k)^{-1} + \textstyle\sum_{i=1}^N [P_i(k)^{-1} - P_i^-(k)^{-1}] \\ \hat{x}(k) &= P(k)[P^-(k)^{-1}\hat{x}^-(k) + \textstyle\sum_{i=1}^N (P_i(k)^{-1}\hat{x}_i(k) - P_i^-(k)^{-1}\hat{x}_i^-(k))] \end{aligned} \tag{42}$$

It is noted that again the global state update equation in the above distributed filter can be written in terms of the information state vector and of the information matrix

$$\begin{aligned} \hat{y}(k) &= \hat{y}^-(k) + \textstyle\sum_{i=1}^N (\hat{y}_i(k) - \hat{y}_i^-(k)) \\ \hat{Y}(k) &= \hat{Y}^-(k) + \textstyle\sum_{i=1}^N (\hat{Y}_i(k) - \hat{Y}_i^-(k)) \end{aligned} \tag{43}$$

The local filters provide their own local estimates and repeat the cycle at step $k+1$. In turn the global filter can predict its global estimate and repeat the cycle at the next time step $k+1$ when the new state $\hat{x}(k+1)$ and the new global covariance matrix $P(k+1)$ are calculated. From Eq. (59) it can be seen that if a local filter (processing station) fails, then the local covariance matrices and the local state estimates provided by the rest of the filters will enable an accurate computation of the system's state vector.

4.3 Local estimations in terms information contributions for the derivative-free EIF

After applying the transformation described in Section 3, the nonlinear discrete-time model of the dynamical system given in Eq. (15) can be substituted by a linear model of the form given in Eq. (1). For this linearized model, the Information Filter (IF) performs fusion of the local state vector estimates which are provided by the local Kalman Filters, using again the *Information matrix* and the *Information state vector* (Rao & Durrant-Whyte, 1991). In place of the Jacobian matrix J_ϕ matrix A_d is used, (discretized equivalent of matrix A_c, which appears in Eq. (18)), while in place of the Jacobian matrix J_γ, matrix C_d is used (discretized equivalent of matrix C_c, which appears in Eq. (19)). As defined before, the Information Matrix is the inverse of the state vector covariance matrix, and can be also associated to the Fisher Information matrix (Rigatos & Zhang, 2009). The Information state vector is the product between the Information matrix and the local state vector estimate

$$\begin{aligned} Y(k) &= P^{-1}(k) = I(k) \\ \hat{y}(k) &= P^-(k)^{-1}\hat{x}(k) = Y(k)\hat{x}(k) \end{aligned} \tag{44}$$

The update equation for the Information Matrix and the Information state vector are given by

$$\begin{aligned} Y(k) &= P^-(k)^{-1} + {C_d}^T(k)R^{-1}(k)C_d(k) \\ &= Y^-(k) + I(k) \end{aligned} \tag{45}$$

$$\begin{aligned} \hat{y}(k) &= \hat{y}^-(k) + {C_d}^T(k)R(k)^{-1}[z(k) - \gamma(x(k)) + C_d\hat{x}^-(k)] \\ &= \hat{y}^-(k) + i(k) \end{aligned} \tag{46}$$

where

$$\begin{aligned} &I(k) = C_d^T(k)R(k)^{-1}C_d(k) \text{ is the associated information matrix and} \\ &i(k) = C_d^T(k)R(k)^{-1}[(z(k) - C_d(k)x(k)) + C_d\hat{x}^-(k)] \text{ is the information state contribution} \end{aligned} \tag{47}$$

The predicted information state vector and Information matrix are obtained from

$$\begin{aligned} \hat{y}^-(k) &= P^-(k)^{-1}\hat{x}^-(k) \\ Y^-(k) = P^-(k)^{-1} &= [A_d(k)P^-(k)A_d(k)^T + Q(k)]^{-1} \end{aligned} \tag{48}$$

The derivative-free Extended Information Filter is next formulated for the case that multiple local sensor measurements and local estimates are used to increase the accuracy and reliability of the estimation. It is assumed that an observation vector $z^i(k)$ is available for N different sensor sites $i = 1, 2, \cdots, N$ and each sensor observes a common state according to the local observation model, expressed by

$$z^i(k) = C_d(k)x(k) + v^i(k),\ i = 1, 2, \cdots, N \tag{49}$$

where the local noise vector $v^i(k) \sim N(0, R^i)$ is assumed to be white Gaussian and uncorrelated between sensors. The variance of a composite observation noise vector v_k is expressed in terms of the block diagonal matrix

$$R(k) = diag[R^1(k), \cdots, R^N(k)]^T \tag{50}$$

The information contribution can be expressed by a linear combination of each local information state contribution i^i and the associated information matrix I^i at the i-th sensor site

$$\begin{array}{c} i(k) = \sum_{i=1}^N {C_d^i}^T(k)R^i(k)^{-1}[z^i(k) - C_d^i(x(k)) + C_d^i(k)\hat{x}^-(k)] \\ I(k) = \sum_{i=1}^N {C_d^i}^T(k)R^i(k)^{-1}C_d^i(k) \end{array} \tag{51}$$

Using Eq. (34) the update equations for fusing the local state estimates become

$$\begin{array}{c} \hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^N {J_\gamma^i}^T(k)R^i(k)^{-1}[z^i(k) - C_d(k)(x(k)) + C_d^i(k)\hat{x}^-(k)] \\ Y(k) = Y^-(k) + \sum_{i=1}^N {C_d^i}^T(k)R^i(k)^{-1}C_d^i(k) \end{array} \tag{52}$$

It is noted that, as in the Extended Information Filter case, an aggregation (master) fusion filter produces a global estimate by using the local sensor information provided by each local filter. The local filters which constitute the Information Filter can be written in terms of *time update* and a *measurement update* equation.

Measurement update: Acquire $z(k)$ and compute

$$\begin{array}{c} Y(k) = P^-(k)^{-1} + C_d^T(k)R(k)^{-1}C_d(k) \\ \text{or } Y(k) = Y^-(k) + I(k) \text{ where } I(k) = C_d^T(k)R^{-1}(k)C_d(k) \end{array} \tag{53}$$

$$\begin{array}{c} \hat{y}(k) = \hat{y}^-(k) + C_d^T(k)R(k)^{-1}[z(k) - C_d(\hat{x}(k)) + C_d\hat{x}^-(k)] \\ \text{or } \hat{y}(k) = \hat{y}^-(k) + i(k) \end{array} \tag{54}$$

Time update: Compute

$$Y^-(k+1) = P^-(k+1)^{-1} = [A_d(k)P(k)A_d(k)^T + Q(k)]^{-1} \tag{55}$$

$$y^-(k+1) = P^-(k+1)^{-1}\hat{x}^-(k+1) \tag{56}$$

4.4 Derivative-free information filtering for state estimates fusion

The outputs of the local Kalman Filters described in subsection 4.3 are treated as measurements which are fed into the aggregation fusion filter (Rao & Durrant-Whyte, 1991). Then each local filter is expressed by its respective error covariance and estimate in terms of information contributions given in Eq.(48)

$$\begin{array}{c} P_i^{-1}(k) = P_i^-(k)^{-1} + C_d^T(k)R^(k)^{-1}C_d(k) \\ \hat{x}_i(k) = P_i(k)(P_i^-(k)^{-1}\hat{x}_i^-(k)) + C_d^T(k)R(k)^{-1}[z^i(k) - C_d^i(k)x(k) + C_d^i(k)\hat{x}_i^-(k)] \end{array} \tag{57}$$

As explained in subsection 4.2, the local estimates are suboptimal and also conditionally independent given their own measurements. The global estimate and the associated error covariance for the aggregate fusion filter can be rewritten in terms of the computed estimates and covariances from the local filters using the relations

$$\begin{array}{c} C_d^T(k)R(k)^{-1}C_d(k) = P_i(k)^{-1} - P_i^-(k)^{-1} \\ C_d^T(k)R(k)^{-1}[z^i(k) - C_d^i(x(k)) + C_d^i(k)\hat{x}^-(k)] = P_i(k)^{-1}\hat{x}_i(k) - P_i(k)^{-1}\hat{x}_i(k-1) \end{array} \tag{58}$$

For the general case of N local filters $i = 1, \cdots, N$, the distributed filtering architecture is described by the following equations

$$\begin{array}{c} P(k)^{-1} = P^-(k)^{-1} + \sum_{i=1}^N [P_i(k)^{-1} - P_i^-(k)^{-1}] \\ \hat{x}(k) = P(k)[P^-(k)^{-1}\hat{x}^-(k) + \sum_{i=1}^N (P_i(k)^{-1}\hat{x}_i(k) - P_i^-(k)^{-1}\hat{x}_i^-(k))] \end{array} \tag{59}$$

It is noted that, once again, the global state update equation in the above distributed filter can be written in terms of the information state vector and of the information matrix

$$\begin{array}{c} \hat{y}(k) = \hat{y}^-(k) + \sum_{i=1}^N (\hat{y}_i(k) - \hat{y}_i^-(k)) \\ \hat{Y}(k) = \hat{Y}^-(k) + \sum_{i=1}^N (\hat{Y}_i(k) - \hat{Y}_i^-(k)) \end{array} \tag{60}$$

The local filters provide their own local estimates and repeat the cycle at step $k+1$. In turn the global filter can predict its global estimate and repeat the cycle at the next time step $k+1$ when the new state $\hat{x}(k+1)$ and the new global covariance matrix $P(k+1)$ are calculated. From Eq. (59) it can be seen again that if a local filter (processing station) fails, then the local covariance matrices and the local state estimates provided by the rest of the filters will enable an accurate computation of the system's state vector.

5. Distributed nonlinear filtering under random delays and packet drops

5.1 Networked Kalman Filtering for an autonomous system

The structure of networked Kalman Filtering is shown in Fig. 4. The problem of distributed filtering becomes more complicated if random delays and packet drops affect the transmission of information between the sensors and local processing units (filters), or between the local filters and the master filter where the fused state estimate is computed. First, results on the stability of the networked linear Kalman Filter will be presented (Xia et al., 2009). The general state-space form of a linear autonomous time-variant dynamical system is given by

$$x(k) = Ax(k-1) + w(k, k-1) \tag{61}$$

where $x(k) \in R^{m\times 1}$ is the system's state vector, $A \in R^{n\times n}$ is the system's state transition matrix, and $w(k, k-1)$ is the white process noise between time instants k and $k-1$.The sensor measurements are received starting at time instant $k \geq 1$ and are described by the measurement equation

$$z(k) = Cx(k) + v(k) \tag{62}$$

where $C \in R^{p\times m}$, $z(k) \in R^{p\times 1}$ and $v(k)$ is the white measurement noise. Measurements $z(k)$ are assumed to be transmitted over a communication channel.

To denote the arrival or loss of a measurement to the local Kalman Filter, through the communication network, one can use variable $\gamma_k \in \{0, 1\}$, where 1 stands for successful delivery of the packet, while 0 stands for loss of the packet.

Thus, in the case of packet losses, the discrete time Kalman Filter recursion that was described in Eq. (5) (measurement update) is modified as

$$K(k) = \gamma_k P^{-}(k)C^T[CP^{-}(k)C^T + R]^{-1} \tag{63}$$

where $\gamma_k \in \{0, 1\}$. This modification implies that the value of the estimated state vector $\hat{x}(k)$ remains unchanged if the a packet drop occurs, i.e. when $\gamma_k = 0$.
It is assumed that the system $[A, C]$ is observable. Next, the following time sequences $\{\tau_k\}$ and $\{\beta_k\}$ are defined $\tau_1 = \inf\{k : k > 1, \gamma_k = 0\}$. Time τ_1 denotes the first time instant when the transmission over the communication channel is interrupted (loss of connection). On the other hand, time sequence β_k is defined as $\beta_1 = \inf\{k : k > \tau_1, \gamma_k = 1\}$. Time β_k denotes the k-th time instant in which the transmission over the communication channel is restored (reestablishment of connection). Therefore, for time sequences τ_k and β_k it holds $1 < \tau_1 < \beta_1 < \tau_2 < \beta_2 < \cdots < \tau_k < \beta_k < \cdots$.
Thus, 1 is the beginning of tranmission, τ_1 is the time instant at which the connection is lost for the first time, β_1 is the time instant at which the connection is re-established after first interruption, τ_2 is the time instant at which the connection is lost for second time, β_2 is the time instant at which the connection is re-established after second interruption,etc. The following variable is also defined $\beta_k^{-} = \beta_k - 1$, where β_k^{-} is the last time instant in a period of subsequent packet losses. Time β_k^{-} is useful for analyzing the behavior of the Kalman Filter in case of a sequence of packet losses (deterioration of the estimation error covariance matrix). It is noted that in the case of the filtering procedure over the communication network, the sequence of covariance matrices P_{β_k} is stable if $sup_{k>1}E||P_{\beta_k}|| < \infty$ (Xia et al., 2009). Equivalently, it can be stated that the networked system satisfies the condition of *peak covariance stability* (Xia et al., 2009).

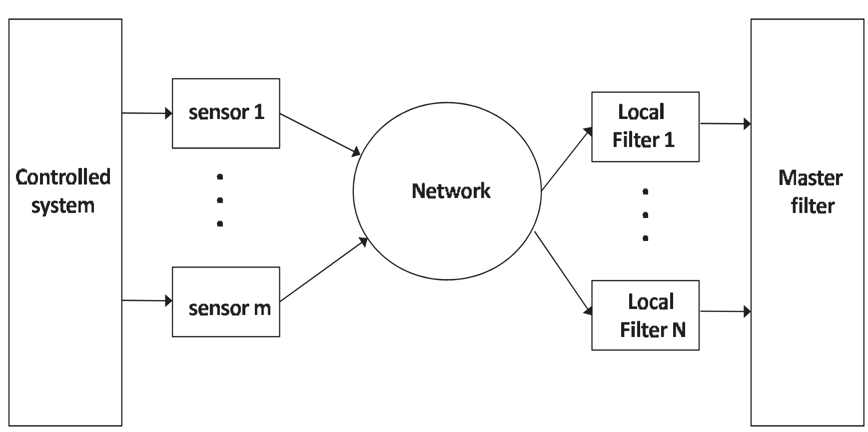


Fig. 4. Distributed filtering over sensors network with communication delays and packet drops

5.2 Processing of the delayed measurements for an autonomous system

Now, the processing of the delayed measurements for the networked linear Kalman Filter proceeds as follows: it is assumed that for all local filters the packet losses and time delays have the same statistical properties. It is also assumed that measurement $z_i(k-N)$ should have arrived at the i-th local filter at time instant $k-N$. Instead of this, the measurement arrives at time instant $k+1$. The delayed measurement $z_i(k-N)$ must be integrated in the estimation which has been performed by each local Kalman Filter (see Fig. 5 and Fig. 6).

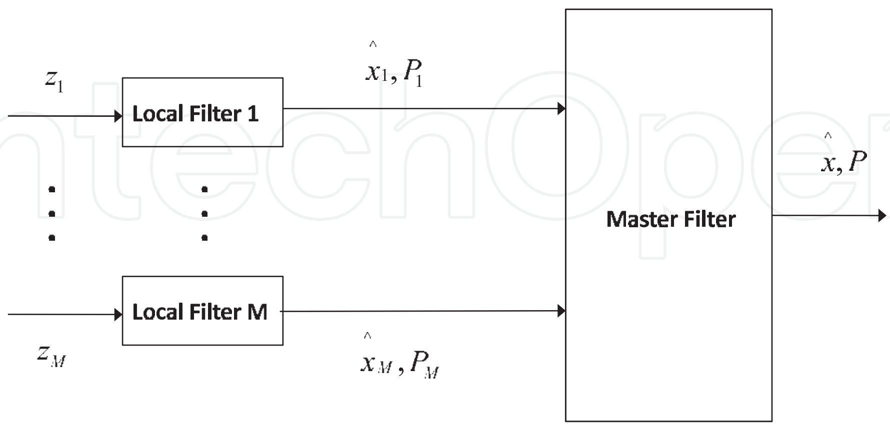


Fig. 5. Distributed filtering diagram implemented with the use of local filters and a master (aggregation) filter

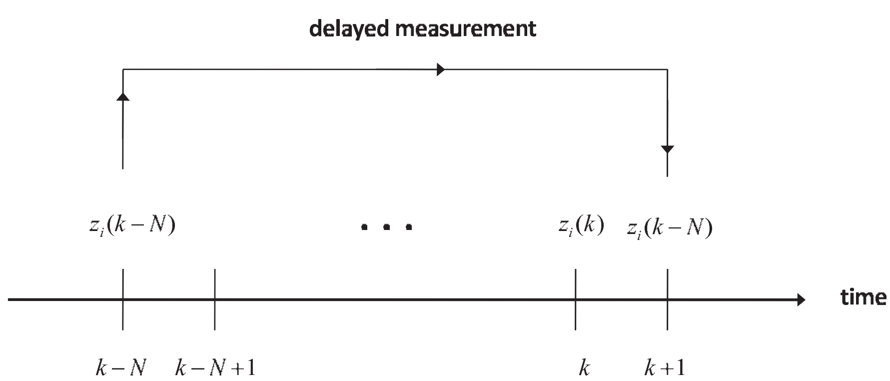


Fig. 6. Delayed measurement over the communication channel

This means that the estimation $\hat{x}_i(k|k)$ and the associated state estimation error covariance matrix $P_i(k|k)$ have to be modified. The transition matrices between different time instances of the discrete-time system of Eq. (61) are defined

$$A(k,k-j) = A(k,k-1)\cdots A(k-j+1,k-j),\ j \in Z^{+} \tag{64}$$

Using the system's dynamic equation in transition matrix form, i.e

$$\begin{aligned} x(k) &= A(k,k-1)x(k-1) + w(k,k-1) \\ z_i(k) &= C_i(k)x(k) + v_i(k) \end{aligned} \tag{65}$$

one has

$$x(k) = A(k,k-N)x(k-N) + w(k,k-N) \tag{66}$$

where

$$w(k,k-N) = \sum_{j=1}^{N} A(k,k-j+1)w(k-j+1,k-j) \quad (67)$$

which means that knowing the state estimation $x(k-N)$ and the sequence of noises from time instant $k-N$ to time instant k one can calculate an estimation of the state vector at time instant k. Denoting $\Phi_1(k-N,k) = A(k,k-N)^{-1}$ and $w_a(k-N,k) = -A(k,k-N)^{-1}w(k,k-N)$ then, from Eq. (66) one obtains

$$x(k-N) = \Phi_1(k-N,k)x(k) + w_a(k-N,k) \quad (68)$$

To incorporate the delayed measurement $z_i(k-N)$ which arrives at the i-th local filter at time instant $k+1$, a state estimation is created first for instant $k-N$ using Eq. (68), i.e.

$$\hat{x}_i(k-N,k) = \Phi_1(k-N,k)\hat{x}_i(k|k) + \hat{w}_a(k-N,k|k) \quad (69)$$

where $\hat{x}_i(k|k)$ is the state estimation of the i-th local filter at time instant k and $\hat{w}_i(k-N,k|k)$ is the noise sequence for the i-th local filter, at time instant $k-N$. For the measurement (output) equation one has from Eq. (65)

$$z_i(k-N) = C_i(k-N)x(k-N) + v_i(k-N) \quad (70)$$

while substituting $x(k-N)$ from Eq. (68) one gets

$$z_i(k-N) = C_i(k-N)\Phi_1(k-N,k)x(k) + C_i(k-N)w_{a_i}(k-N,k) + v_i(k-N) \quad (71)$$

Next, using the current state estimate $\hat{x}(k|k)$ and Eq. (71) one can find the measurement estimate $\hat{z}_i(k-N|k)$ for the i-th local filter, $i = 1, \cdots, M$:

$$\hat{z}_i(k-N) = C_i(k-N)\Phi_{1i}(k-N,k)\hat{x}(k|k) + C_i(k-N)\hat{w}_{\alpha_i}(k-N,k) \quad (72)$$

Defining, $\tilde{z}_i(k|j) = z_i(k) - \hat{z}_i(k|j)$ (innovation), $\tilde{x}_i(k|j) = x(k) - \hat{x}_i(k|j)$ (state estimation error), and $\tilde{w}_i(k-N,k|k) = w(k-N,k) - \hat{w}_i(k-N,k)$ (noise estimation error) one obtains

$$\tilde{z}_i(k-N|k) = C_i(k-N)\Phi_{1i}(k-N,k)\tilde{x}_i(k|k) + C_i(k-N)\tilde{w}_{a_i}(k-N,k|k) + v_i(k-N) \quad (73)$$

The innovation $\tilde{z}_i(k-N,k)$ at time instant $k-N$ will be used to modify the estimation $\hat{x}_i(k|k)$ into

$$\hat{x}_i^*(k|k) = \hat{x}_i(k|k) + M_i\tilde{z}_i(k-N|k) \quad (74)$$

Thus, one can update (smooth) the state estimate at time instant k by adding to the current state estimate $\hat{x}_i(k|k)$ the corrective term

$$M_i\tilde{z}_i(k-N,k) \quad (75)$$

where M_i is a gain matrix to be defined in the sequel, and $\tilde{z}_i(k-N,k)$ is the innovation between the measurement $z_i(k-N)$ taken at time instant $k-N$ and the output estimate $\hat{z}_i(k-N)$ which has been calculated in Eq. (72).
The main difficulty in Eq. (74) is that one has to calculate first the noise estimation error $\tilde{w}_{a_i}(k-N,k|k)$, which means that one has to calculate an estimate of the process noise $\hat{w}_{a_i}(k-N,k)$.

The following theorem has been stated in (Xia et al., 2009), and is also applicable to the distributed filtering approach presented in this chapter:

Theorem 1: It is assumed that the observation error (innovation) at the i-th information processing unit (local filter), at time instant $k-n$ where $n \in [0, N]$, is given by

$$\tilde{z}(k-n) = z_i(k-n) - \hat{z}_i(k-n) \tag{76}$$

and that the covariance matrix of the white process noise $w_a(k-j+1, k-j)$ is

$$Q(k-j+1, k-j) = E\{w_a(k-j+1, k-j) w_a(k-j+1, k-j)^T\} \tag{77}$$

while the estimation error for the noise $w_{ai}(k-N, k|k)$ is

$$\tilde{w}_{a_i}(k-N, k|k) = w_{a_i}(k-N, k) - \hat{w}_{a_i}(k-N, k|k) \tag{78}$$

Moreover, the covariance matrix for the error of the white estimated noise vector $\tilde{w}_{a_i}(k-N, k|k)$ is

$$Q_i^*(k-N, k) = E\{\tilde{w}_{a_i}(k-N, k|k) \tilde{w}_{a_i}(k-N, k|k)^T\} \tag{79}$$

Then, one can obtain the noise estimate $\hat{w}_{a_i}(k-N, k)$ from the relation

$$\begin{gathered} \hat{w}_{a_i}(k-N, k|k) = -\Phi_1(k-N, k) \textstyle\sum_{n=0}^{N-1} \\ \tilde{C}_i(n)[C_i(k-n)P_i(k-n|k-n-1)C_i(k-n)^T + R_i(k-n)]^{-1} \tilde{z}_i(k-n) \end{gathered} \tag{80}$$

where

$$\begin{gathered} \tilde{C}_i(n) = \{A(k, k-n)Q(k-n, k-n-1) + \textstyle\sum_{j=n+2}^{N} A(k, k-j+1)Q(k-j+1, k-j) \times \\ \times [\textstyle\prod_{m=n+1}^{j-1} A(k-m+1, k-m)[I - K_i(k-m)C_i(k-m)]]^T\} C_i(k-n)^T \end{gathered} \tag{81}$$

while the covariance matrix of the estimated white noise $w_{\alpha_i}(k-N, k)$ is calculated as

$$\begin{gathered} Q_i^*(k-N, k) = Q(k-N, k) - \Phi_1(k-N, k) \times \\ \times \textstyle\sum_{n=0}^{N-1} \tilde{C}_i(n)[C_i(k-n)P_i(k-n|k-n-1)C_i(k-n)^T + R_i(k-n)]^{-1} \times \\ \times \tilde{C}_i(n)^T \Phi_1(k-N, k)^T \end{gathered} \tag{82}$$

where

$$\begin{gathered} Q(k-N, k) = \Phi_1(k-N, k)[\textstyle\sum_{j=1}^{N} A(k, k-j+1) \times \\ \times Q(k-j+1, k-j)A(k, k-j+1)^T]\Phi_1(k-N, k)^T \end{gathered} \tag{83}$$

Next, a theorem is given about the calculation of covariance matrix M_i appearing in the modified state estimation of Eq. (74). The theorem comes from (Xia et al., 2009) and is also applicable to the distributed filtering approach which is presented in this chapter.

Theorem 2: It is assumed that the modified state estimation error at time instant k is

$$\tilde{x}_i^*(k|k) = x(k) - \hat{x}_i(k|k) \tag{84}$$

and that the covariance matrix of the modified state estimation error is

$$P_i^*(k|k) = E\{\tilde{x}_i^*(k|k)\tilde{x}_i^*(k|k)^T\} \tag{85}$$

and that the cross-covariance between $\tilde{x}_i(k|k)$ and $\tilde{w}_i(k-N,k|k)$ is

$$P_i^{\tilde{x}\tilde{w}}(k|k) = E\{\tilde{x}_i(k|k)\tilde{w}_i(k-N,k|k)^T\} \tag{86}$$

Then, the optimal filter for the processing of the delayed measurements is given by Eq. (74), i.e.

$$\hat{x}_i^*(k|k) = \hat{x}_i(k|k) + M_i[z_i(k-N) - \hat{z}_i(k-N|k)] \tag{87}$$

where

$$M_i = [P_i(k|k)\Phi_1(k-N,k)^T + P_i^{\tilde{x}\tilde{w}}]C_i(k-N)^T W_i^{-1} \tag{88}$$

In that case, the covariance matrix of the modified state estimation error becomes

$$\begin{array}{c} P_i^*(k|k) = P_i(k|k) - [P_i^{\tilde{x}\tilde{w}} + P_i(k|k)\Phi_1(k-N,k)^T]\times \\ \times C_i(k-N)^T W_i^{-1} C_i(k-N)\times \\ \times[P_i^{\tilde{x}\tilde{w}} + P_i(k|k)\Phi_1(k-N,k)^T]^T \end{array} \tag{89}$$

where matrices W_i and $P_i^{\tilde{x}\tilde{w}}$ are defined as

$$\begin{array}{c} W_i = C_i(k-N)\{\Phi_1(k-N,k)P_i(k|k)\Phi_1(k-N,k)^T + \\ +\Phi_1(k-N,k)P_i^{\tilde{x}\tilde{w}} + [A(k-N,k)P_i^{\tilde{x}\tilde{w}}]^T + Q_i^*(k-N,k)\} \\ \times C_i(k-N)' + R_i(k-N) \end{array} \tag{90}$$

$$\begin{array}{c} P_i^{\tilde{x}\tilde{w}} = \Phi_1(k-N,k)\sum_{n=0}^{N-1} P_i(k-N|k-N)D_i(n)^T\times \\ \times[C_i(k-n)P_i(k-n|k-n-1)C_i(k-n)^T + R_i(k-n)]^{-1}\times \\ \times\tilde{C}_i(n)^T\Phi_1(k-N,k)^T - A(k,k-N)Q_i^*(k-N,k) \end{array} \tag{91}$$

and matrix $D_i^T(n)$ is defined as

$$D_i(n) = \begin{cases} C_i(k-n)A(k-n,k-n-1), \text{ if } N=1 \\ C_i(k-n)A(k-n,k-n-1)\prod_{j=n}^{N-2}[I - K_i(k-j-1)C_i(k-j-1)]\times \\ \times A(k-j-1,k-j-2), \text{ if } N>1 \end{cases} \tag{92}$$

5.3 Processing of the delayed measurements for a linear non-autonomous system

5.3.1 The case of a time-variant linear system

In the case of a linear non-autonomous system, in place of Eq. (61) one has

$$x(k) = A(k,k-1)x(k-1) + B(k,k-1)u(k-1) + w(k,k-1) \tag{93}$$

Setting $w_1(k,k-1) = B(k,k-1)u(k-1) + w(k,k-1)$ one obtains

$$x(k) = A(k,k-1)x(k-1) + w_1(k,k-1) \tag{94}$$

and consequently it holds

$$\begin{array}{c} x(k) = \prod_{j=1}^{N} A(k-j+1,k-j)x(k-N) + \\ +\sum_{m=1}^{N-1}\prod_{j=1}^{m} A(k-j+1,k-j)w_1(k-m,k-m-1) + w_1(k,k-1) \end{array} \tag{95}$$

where

$$w_1(k-m+1,k-m) = B(k-m+1,k-m)u(k-m) + w(k-m+1,k-m) \quad (96)$$

Thus, one can obtain a more compact form

$$x(k) = \Phi(k,k-N)x(k-N) + w_1(k,k-N) \quad (97)$$

with

$$\begin{array}{c} \Phi(k,k-N) = \prod_{j=1}^{N} A(k-j+1,k-j), \text{ and} \\ w_1(k,k-N) = \sum_{m=1}^{N-1}\prod_{j=1}^{m} A(k-j+1,k-j)w_1(k-m,k-m-1) + w_1(k,k-1) \end{array} \quad (98)$$

5.3.2 The case of a time-invariant linear system

For a linear time-invariant non-autonomous system

$$x(k) = Ax(k-1) + Bu(k-1) + w(k-1) \quad (99)$$

it holds

$$x(k) = A^N x(k-N) + \sum_{j=1}^{N} A^{N-j}Bu(k-N+j-1) + \sum_{j=1}^{N} A^{N-j}w(k-N+j-1) \quad (100)$$

Denoting $A^N = \Phi(k,k-N)$ one has

$$x(k) = \Phi(k,k-N)x(k-N) + \sum_{j=1}^{N} A^{N-j}Bu(k-N+j-1) + \sum_{j=1}^{N} A^{N-j}w(k-N+j-1) \quad (101)$$

Setting

$$w_1(k,k-N) = \sum_{j=1}^{N} A^{N-j}Bu(k-N+j-1) + \sum_{j=1}^{N} A^{N-j}w(k-N+j-1) \quad (102)$$

one has that Eq. (101) can be written in a more compact form as

$$x(k) = \Phi(k,k-N)x(k-N) + w_1(k,k-N) \quad (103)$$

Using that matrix $\Phi(k,k-N)$ is invertible, one has

$$x(k-N) = \Phi(k,k-N)^{-1}x(k) - \Phi(k,k-N)^{-1}w_1(k,k-N) \quad (104)$$

The following notation is used $\Phi_1(k-N,k) = \Phi(k,k-N)^{-1}$ while for the retrodiction of $w_1(k,k-N)$ it holds $w_a(k-N,k|k) = -\Phi(k,k-N)^{-1}w_1(k,k-N)$. Then, to smooth the state estimation at time instant $k-N$, using the measurement of output $z_i(k-N)$ received at time instant $k+1$ one has the state equation

$$\hat{x}(k-N,k) = \Phi_1(k-N,k)\hat{x}(k|k) + \hat{w}_a(k-N,k|k) \quad (105)$$

while the associated measurement equation becomes

$$z(k-N) = Cx(k-N) + v(k-N) \tag{106}$$

Substituting Eq. (104) into Eq. (106) provides

$$z(k-N) = C\Phi_1(k-N,k)x(k) + Cw_a(k-N,k) + v(k-N) \tag{107}$$

and the associated estimated-output at time instant $k-N$ is

$$\hat{z}(k-N) = C\Phi_1(k-N,k)\hat{x}(k|k) + C\hat{w}_a(k-N,k) \tag{108}$$

From Eq. (108) and Eq. (107) the innovation for the delayed measurement can be obtained

$$\tilde{z}(k-N) = z(k-N) - \hat{z}(k-N) \tag{109}$$

i.e. $\tilde{z}(k-N) = C\Phi_1(k-N)\tilde{x}(k|k) + C\tilde{w}_a(k-N)$, where $\tilde{x}(k|j) = x(k) - \hat{x}(k|j)$ is the state estimation error and $\hat{w}_a(k-N,k|k) = w_a(k-N,k) - \hat{w}_a(k-N,k)$ is the estimation error for w_α. With this innovation the estimation of the state vector $x(k|k)$ at time instant k is corrected. The correction (smoothing) relation is

$$\hat{x}^*(k|k) = x(k|k) + M\tilde{z}(k-N,k) \tag{110}$$

Therefore, again the basic problem for the implementation of the smoothing relation provided by Eq. (110) is the calculation of the term $w_a(k-N,k)$ i.e. $w_a(k-N) = \Phi(k,k-N)^{-1}w_1(k,k-N)$. This in turn requires the estimation of the term $w_1(k-N,k)$ which, according to Eq. (80), is provided by

$$\begin{array}{c} \hat{w}_1(k-N,k) = -\Phi_1(k-N,k)\sum_{n=0}^{N-1} \cdot \\ \cdot\tilde{C}(n)[C(k-n)P(k-n|k-n-1)C(k-n)]^T + R(k-n)]^{-1}\tilde{z}(k-n) \end{array} \tag{111}$$

where $\tilde{z}(k-n) = z(k-n) - \hat{z}(k-n)$ is the innovation for time-instant $k-n$, while, as given in Eq. (81)

$$\begin{array}{c} \tilde{C}(n) = \{A(k,k-n)Q(k-n,k-n-1) + \sum_{j=n+2}^{N} A(k,k-j+1)Q(k-j+1,k-j)\times \\ \times[\prod_{m=n+1}^{j-1} A(k-m+1,k-m)[I-K_i(k-m)C(k-m)]^T]\}C(k-n)^T \end{array} \tag{112}$$

5.4 Derivative-free Extended Information Filtering under time-delays and packet drops

It has been shown that using a suitable transform (diffeomorphism), the nonlinear system of Eq. (15) can be transformed into the system of Eq. (17). Moreover, it has been shown that for the systems of Eq. (23) and Eq. (24) one can obtain a a state-space equation of the form

$$\begin{pmatrix} \dot{\zeta}_1 \\ \dot{\zeta}_2 \\ \cdots \\ \dot{\zeta}_{n-1} \\ \dot{\zeta}_n \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \cdots \\ \zeta_{n-1} \\ \zeta_n \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \cdots \\ 0 \\ 1 \end{pmatrix} v(\zeta,t) \tag{113}$$

$$z = \begin{pmatrix} 1 & 0 & 0 & \cdots & 0 \end{pmatrix} \zeta \tag{114}$$

where $v(t) = f(x,t) + g(x,t)u(t)$, with $u(t)$ being the control input of the dynamical system. The description of the initial system of Eq. (17) in the form of Eq. (113) and Eq. (114) enables

the application of the previous analysis for the compensation of time-delays and packet-drops through smoothing in the computation of the linear Kalman Filter. The fact that the system of Eq. (113) and Eq. (114) is a time invariant one, facilitates the computation of the smoothing Kalman Filter given in Eq. (100) to Eq. (112). Thus, one has to use the time invariant matrices A_c and C_c defined in Eq. (18) and Eq. (19), while for matrix B_c it holds according to Eq. (25) that $B_c = [0, 0, \cdots, 0, 1]^T$. The discrete-time equivalents of matrices A_c, B_c and C_c are noted as A_d, B_d and C_d, respectively. It is also noted that due to the specific form of matrix B_c, the term $Bu(k-1)$ appearing in Eq. (99) is a variable of small magnitude with mean value close to zero. Thus the term $w_1(k, k-1) = Bu(k-1) + w(k, k-1)$ differs little from $w(k, k-1)$.
It also becomes apparent that through the description of the initial system of Eq. (17) in the form of Eq. (113) and Eq. (114), the application of the derivative-free Extended Information Filter can be performed in a manner that enables the compensation of time-delays and packet drops. Writing the controlled system in the form of Eq. (113) and Eq. (114) permits to develop local linear Kalman Filters that smooth the effects of delayed sensor measurement or the loss of measurement packets. Moreover, the application of the standard Information Filter for fusing the estimates provided by the local Kalman Filters, permits to avoid the approximation errors met in the Extended Information Filter algorithm.

6. Distributed filtering under time-delays and packet drops for sensorless control

6.1 Visual servoing over a network of synchronized cameras

Visual servoing over a network of synchronized cameras is an example where the efficiency of the proposed distributed filtering approach under time delays and packet drops can be seen. Applications of vision-based robotic systems are rapidly expanding due to the increase in computer processing power and low prices of cameras, image grabbers, CPUs and computer memory. In order to satisfy strict accuracy constraints imposed by demanding manufacturing specifications, visual servoing systems must be fault tolerant. This means that despite failures in its components or the presence of disturbances, the system must continue to provide valid control outputs which will allow the robot to complete its assigned tasks (DeSouza & Kak, 2004),(Feng & Zeng, 2010),(Hwang & Shih, 2002),(Malis et al., 2000).
The example to be presented describes the control of a planar robot with the use of a position-based visual servo that comprises multiple fixed cameras. The chapter's approach relies on neither position nor velocity sensors, and directly sets the motor control current using only visual feedback. Direct visual servoing is implemented using a distributed filtering scheme which permits to fuse the estimates of the robot's state vector computed by local filters, each one associated to a camera in the cameras network (see Fig. 7). The cameras' network can be based on multiple RS-170 cameras connected to a computer with a frame grabber to form a vision node. Each vision node consists of the camera, the frame grabber and the filter which estimates motion characteristics of the monitored robot joint. The vision nodes are connected in a network to form a distributed vision system controlled by a master computer. The master computer is in turn connected to a planar 1-DOF robot joint and uses the vision feedback to perform direct visual servoing (see Fig. 7).
The master computer communicates video synchronization information over the network to each vision node. Typical sources of measurement noise include charge-coupled device (CCD) noise, analog-to-digital (A/D) noise and finite word-length effects. Under ideal conditions, the effective noise variance from these sources should remain relatively constant. Occlusions can be also considered as a noise source. Finally, communication delays and packet drops in the transmission of measurements from the vision sensors to the information processing

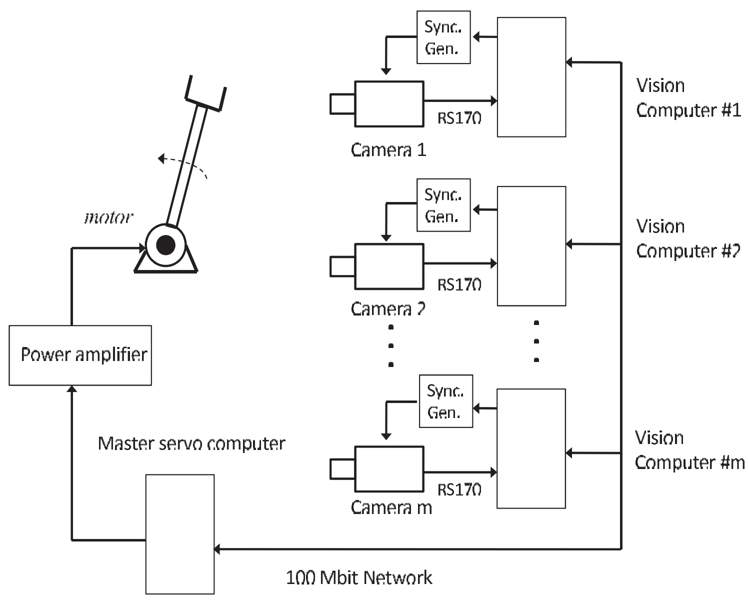


Fig. 7. Distributed cameras network and distributed information processing units for visual servoing

nodes induce additional disturbances which should be compensated by the virtual servoing control loop.

6.2 Distributed filtering-based fusion of the robot's state estimates

Fusion of the local state estimates which are provided by filters running on the vision nodes can improve the accuracy and robustness of the performed state estimation, thus also improving the performance of the robot's control loop (Sun et al., 2011),(Sun & Deng, 2005). Under the assumption of Gaussian noise, a possible approach for fusing the state estimates from the distributed local filters is the derivative-free Extended Information Filter (DEIF). As explained in Section 4, the derivative-free Extended Information Filter provides an aggregate state estimate by weighting the state vectors produced by local Kalman Filters with the inverse of the associated estimation error covariance matrices.

Visual servoing over the previously described cameras network is considered for the nonlinear dynamic model of a single-link robotic manipulator. The robot can be programmed to execute a manufacturing task, such as disassembly or welding (Tzafestas et al., 1997). The position of the robot's end effector in the cartesian space (and consequently the angle for the robotic link) is measured by the aforementioned m distributed cameras. The proposed multi-camera based robotic control loop can be also useful in other vision-based industrial robotic applications where the vision is occluded or heavily disturbed by noise sources, e.g. cutting. In such applications there is need to fuse measurements from multiple cameras so as to obtain redundancy in the visual information and permit the robot to complete safely and within the specified accuracy constraints its assigned tasks (Moon et al, 2006),(Yoshimoto et al., 2010).

The considered 1-DOF robotic model consists of a rigid link which is rotated by a DC motor, as shown in Fig. 8. The model of the DC motor is described by the set of equations: $L\dot{I} = -k_e\omega - RI + V$, $J\dot{\omega} = k_e I - k_d\omega - \Gamma_d$, with the following notations L : armature inductance, I : armature current, k_e : motor electrical constant, R : armature resistance, V : input voltage,

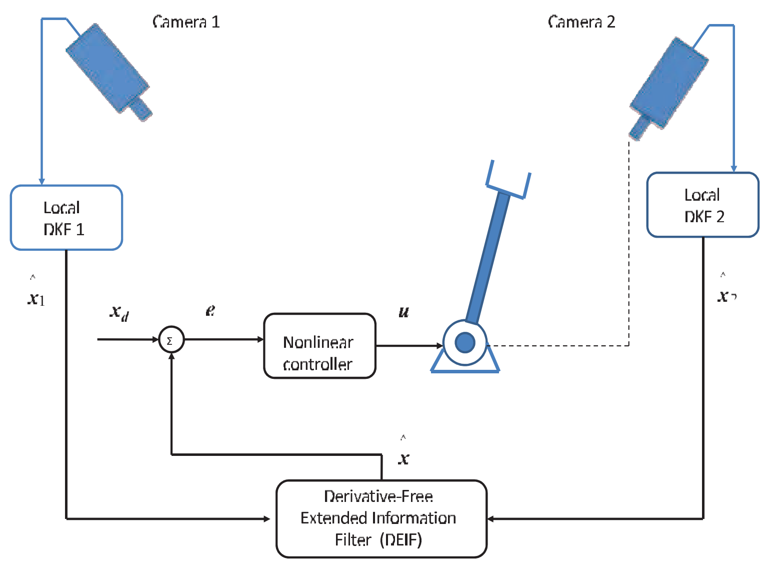


Fig. 8. Visual servoing based on fusion of state estimates provided by local derivative-free nonlinear Kalman Filters

taken as control input, J : motor inertia, ω : rotor rotation speed, k_d : mechanical dumping constant, Γ_d : disturbance or external load torque. It is assumed that $\Gamma_d = mgl{\cdot}sin(\theta)$, i.e. that the DC motor rotates a rigid robotic link of length l with a mass m attached to its end. Then, denoting the state vector as $[x_1, x_2, x_3]^T = [\theta, \dot{\theta}, \ddot{\theta}]^T$, a nonlinear model of the DC motor is obtained

$$\dot{x} = f(x,t) + g(x,t)u \tag{115}$$

where $f(x,t) = [f_1(x,t), f_2(x,t), f_3(x,t)]^T$ is a vector field function with elements: $f_1(x,t) = x_2$, $f_2(x,t) = x_3$, $f_3(x,t) = -\frac{k_e^2+k_dR}{JL}x_2 - \frac{RJ+K_dL}{JL}x_3 - \frac{Rmgl}{JL}sin(x_1) - \frac{mgl}{J}cos(x_1)x_2$. Similarly, for function $g(x,t)$ it holds that $g(x,t) = [g_1(x,t), g_2(x,t), g_3(x,t)]^T$, i.e. it is a vector field function with elements: $g_1(x,t) = 0$, $g_2(x,t) = 0$, $g_3(x,t) = \frac{k_e}{JL}$. Having chosen the joint's angle to be the system's output, the state space equation of the 1-DOF robot manipulator can be rewritten as

$$x^{(3)} = \bar{f}(x) + \bar{g}(x)u \tag{116}$$

where functions $\bar{f}(x)$ and $\bar{g}(x)$ are given by $\bar{f}(x) = -\frac{k_e^2+k_dR}{JL}x_2 - \frac{RJ+K_dL}{JL}x_3 - \frac{Rmgl}{JL}sin(x_1) - \frac{mgl}{J}cos(x_1)x_2$, and $\bar{g}(x) = \frac{k_e}{JL}$. This is a system in the form of Eq. (23), therefore a state estimator can be designed according to the previous results on derivative-free Kalman Filtering.

The controller has to make the system's output (angle θ of the motor) follow a given reference signal x_d. For measurable state vector x and uncertain functions $f(x,t)$ and $g(x,t)$ an appropriate control law for the 1-DOF robotic model is

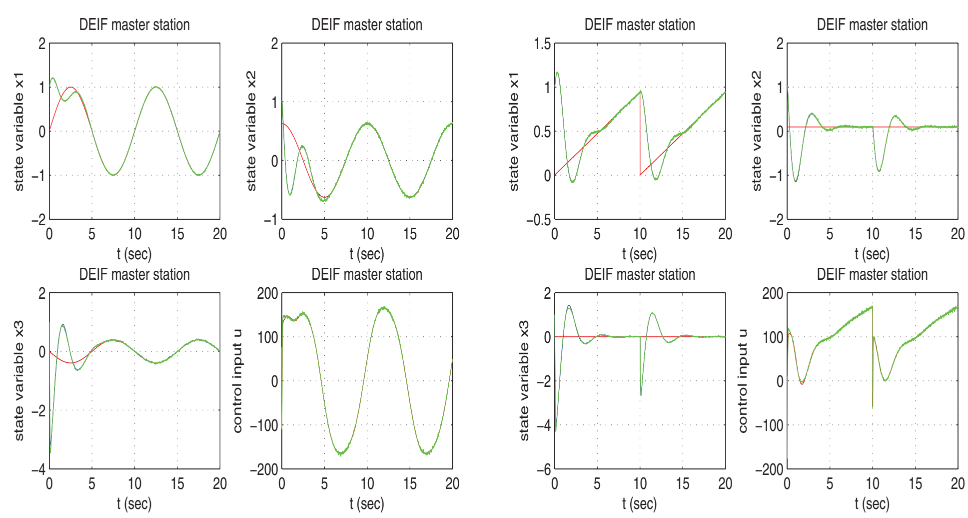
$$u = \frac{1}{g(x,t)}[x_d^{(n)} - f(x,t) - K^Te + u_c] \tag{117}$$

with $e = x - x_d$, $e^T = [e, \dot{e}, \ddot{e}, \cdots, e^{(n-1)}]^T$, $K^T = [k_n, k_{n-1}, \cdots, k_1]$, such that the polynomial $e^{(n)} + k_1 e^{(n-1)} + k_2 e^{(n-2)} + \cdots + k_n e$ is Hurwitz. The previously defined control law results into $e^{(n)} = -K^T e + u_c + \tilde{d}$, where the supervisory control term u_c aims at the compensation of modeling errors as well as of the additive disturbance $\tilde{d}$ (Rigatos & Tzafestas, 2007). Suitable selection of the feedback gain K assures that the tracking error will converge to $\lim_{t\to\infty} e(t) = 0$. In case of state estimation-based (sensorless control), and denoting, $\hat{x}$ as the estimated state vector and $\hat{e} = \hat{x} - x_d$ as the estimated tracking error one has

$$u = \frac{1}{g(\hat{x}, t)} [x_d^{(n)} - f(\hat{x}, t) - K^T \hat{e} + u_c] \tag{118}$$

7. Simulation tests

The fusion of the distributed state estimates for the robotic model was performed with the use of the derivative-free Extended Information Filter. First, it was assumed that the transmission of measurements from the vision sensors (cameras) to the local information processing units, where the state estimators (filters) were running, was not affected by time delays or packet drops. At the local vision nodes, Kalman filters were used to produce estimations of the robot's state vector as well as the associated covariance matrices, after carrying out a linearization of the robot's nonlinear dynamic model through the transformation described in subsection 3.2 and processing the local xy position measurements. This standard Information Filter provided the overall estimate of the robot's state vector, through weighting of the local state vectors by the local covariance matrices. The obtained results are depicted in Fig. 9(a) and Fig. 9(b) in case of a sinusoidal and a see-saw reference trajectory (both reference trajectories are denoted with the red line).



(a) **(b)**

Fig. 9. Control of the robotic manipulator with fusion of position measurements from distributed cameras through the use of the derivative-free Extended Information Filter (a) when tracking of a sinusoidal trajectory (b) when tracking of a see-saw trajectory

Next, time-delays were assumed in the transmission of image frames from the distributed cameras to the associated local vision nodes, where the local derivative-free Kalman Filters were running. For both vision nodes the delays in the transmission of measurements varied randomly between 6 and 25 sampling periods. Longer delays could be also handled by the proposed distributed filtering algorithm. The variation of measurement transmission delays with respect to time, is depicted in Fig. 10.

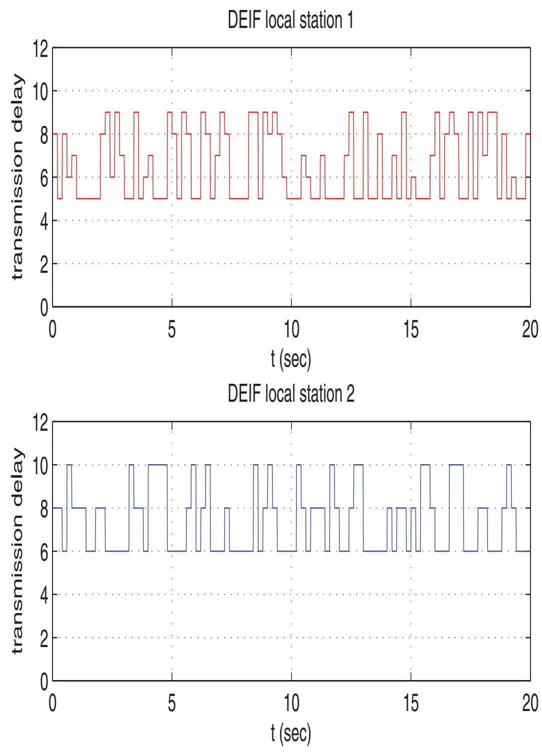
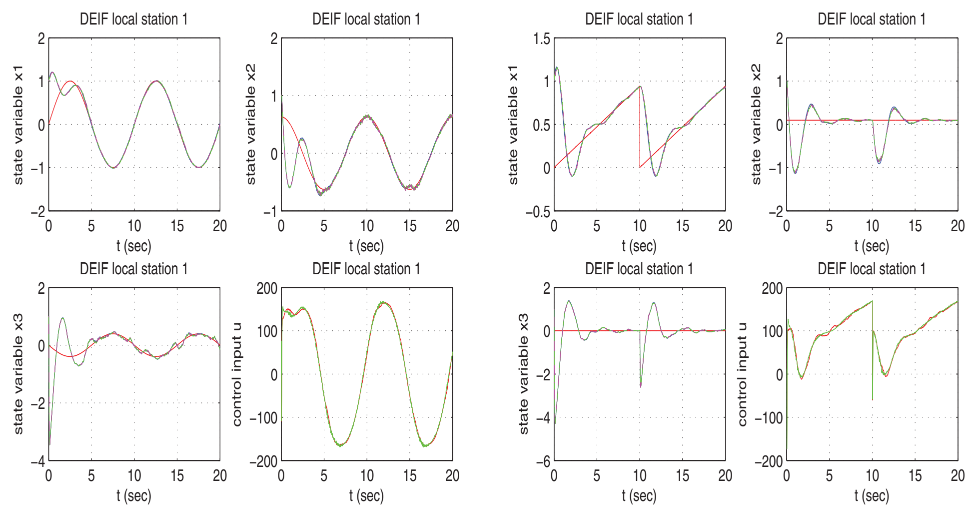


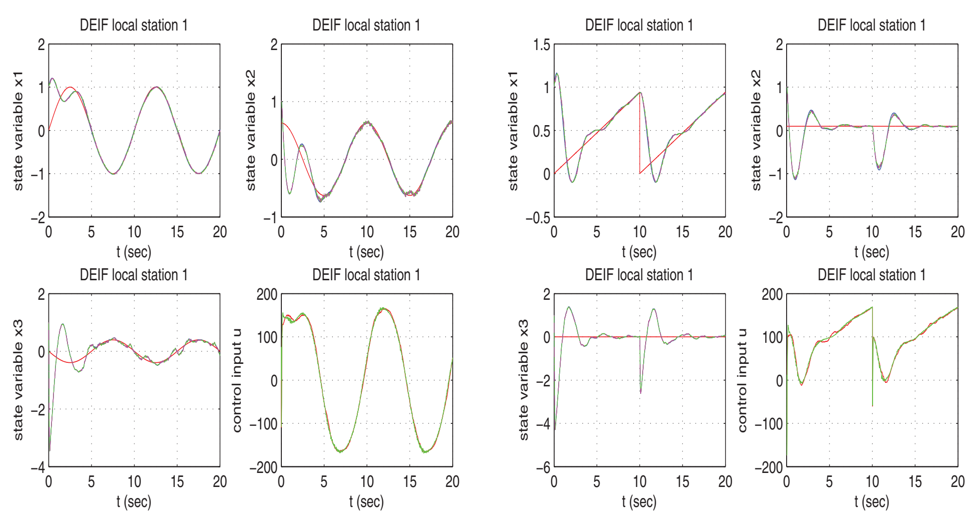
Fig. 10. Variation in time (in multiples of the sampling period) of the measurement delays appearing at the local information processing nodes 1 and 2.

The delayed measurements were processed by the Kalman Filter recursion according to the stages explained in subsection 5.3. The smoothing of the delayed measurements that was performed by the Kalman Filter was based on Eq. (74), i.e. $\hat{x}^*(k|k) = x(k|k) + M\tilde{y}(k-N,k)$. As explained in subsection 5.3, matrix M is a gain matrix calculated according to Eq. (88). The innovation is given by $\tilde{z}(k-N) = z(k-N) - \hat{z}(k-N)$. The tracking accuracy of the distributed filtering-based control loop is depicted in Fig. 11 to Fig. 13.
Additionally, some performance metrics were used to evaluate the distributed filtering-based control scheme. Table I, shows the variation of the traces of the covariance matrices at the local filters and at the master filter with respect to delay levels ($d_1, d_2 = k{\cdot}T_s$ i.e. multiples of the sampling period T_s), as well as with respect to the probability of delay occurrence in the transmission of the measurement packets ($p \in [0,1]$).
Moreover, the variation of the tracking error of the three state variables x_i, $i = 1, \cdots, 3$ with respect to delay levels as well as with respect to the probability of delay occurrence in the transmission of the measurement packets is given in Tables II to IV.



(a) **(b)**

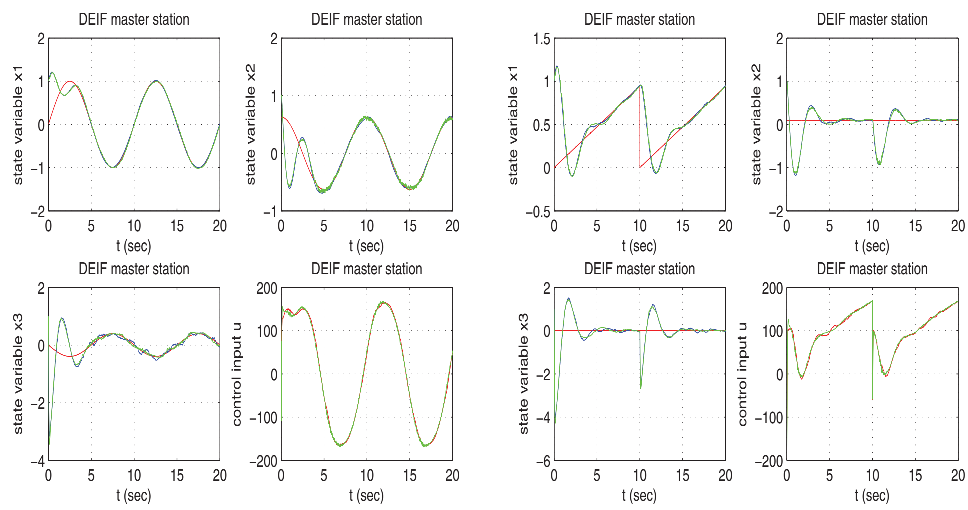
Fig. 11. Estimation of the motion of the robotic manipulator under transmission delays at the first local measurement processing node, (a) when tracking a sinusoidal trajectory (b) when tracking a see-saw trajectory



(a) **(b)**

Fig. 12. Estimation of the motion of the robotic manipulator under transmission delays at the second local measurement processing node, (a) when tracking a sinusoidal trajectory (b) when tracking a see-saw trajectory

It can be noticed that the smoothing performed by the distributed filtering algorithm, through the incorporation of out-of-sequence-measurements, enhances the robustness of the



(a) **(b)**

Fig. 13. Control of the robotic manipulator under measurement transmission delays and using the derivative-free Extended Information Filter for state estimation, (a) tracking of a sinusoidal trajectory (b) tracking of a see-saw trajectory

Table I: Traces of the covariance error matrices for various delay levels

d_1	d_2	p	$Tr(P_1^*)$	$Tr(P_2^*)$	$Tr(P)$
0	0	0.0	$2.780 \cdot 10^{-2}$	$2.780 \cdot 10^{-2}$	$6.720 \cdot 10^{-3}$
6	8	0.8	$2.782 \cdot 10^{-2}$	$2.777 \cdot 10^{-2}$	$6.730 \cdot 10^{-3}$
9	10	0.8	$2.782 \cdot 10^{-2}$	$2.783 \cdot 10^{-2}$	$6.730 \cdot 10^{-3}$
12	15	0.8	$2.782 \cdot 10^{-2}$	$2.776 \cdot 10^{-2}$	$6.730 \cdot 10^{-3}$
18	20	0.6	$2.782 \cdot 10^{-2}$	$2.775 \cdot 10^{-2}$	$6.730 \cdot 10^{-3}$
25	30	0.6	$2.780 \cdot 10^{-2}$	$2.783 \cdot 10^{-2}$	$6.730 \cdot 10^{-3}$

Table II: RMSE tracking error at the 1st local filter for various delay levels

d_1	d_2	p	$x_1 - x_1^d$	$x_2 - x_2^d$	$x_3 - x_3^d$
0	0	0.0	$4.419 \cdot 10^{-3}$	$5.490 \cdot 10^{-3}$	$1.125 \cdot 10^{-2}$
6	8	0.8	$4.413 \cdot 10^{-3}$	$5.504 \cdot 10^{-3}$	$1.129 \cdot 10^{-2}$
9	10	0.8	$4.392 \cdot 10^{-3}$	$5.437 \cdot 10^{-3}$	$1.121 \cdot 10^{-2}$
12	15	0.8	$4.402 \cdot 10^{-3}$	$5.465 \cdot 10^{-3}$	$1.117 \cdot 10^{-2}$
18	20	0.6	$4.474 \cdot 10^{-3}$	$5.707 \cdot 10^{-3}$	$1.151 \cdot 10^{-2}$
25	30	0.6	$4.433 \cdot 10^{-3}$	$5.655 \cdot 10^{-3}$	$1.144 \cdot 10^{-2}$

estimation. Despite the raise of the delay levels in the transmission of measurements from the sensors (cameras) to the local information processing nodes (local derivative-free Kalman Filters) only slight variations of the tracking errors for state variables x_i, $i = 1, \cdots, 3$ were observed. Similarly, the changes of the traces of the estimation error covariance matrices, both at the local filters and at the master filter, were small.

Table III: RMSE tracking error at the 2nd local filter for various delay levels

d_1	d_2	p	$x_1 - x_1^d$	$x_2 - x_2^d$	$x_3 - x_3^d$
0	0	0.0	$4.390 \cdot 10^{-3}$	$5.453 \cdot 10^{-3}$	$1.116 \cdot 10^{-2}$
6	8	0.8	$4.380 \cdot 10^{-3}$	$5.468 \cdot 10^{-3}$	$1.114 \cdot 10^{-2}$
9	10	0.8	$4.441 \cdot 10^{-3}$	$5.495 \cdot 10^{-3}$	$1.118 \cdot 10^{-2}$
12	15	0.8	$4.451 \cdot 10^{-3}$	$5.521 \cdot 10^{-3}$	$1.125 \cdot 10^{-2}$
18	20	0.6	$4.508 \cdot 10^{-3}$	$5.744 \cdot 10^{-3}$	$1.161 \cdot 10^{-2}$
25	30	0.6	$4.432 \cdot 10^{-3}$	$5.755 \cdot 10^{-3}$	$1.150 \cdot 10^{-2}$

Table IV: RMSE tracking error at the master filter for various delay levels

d_1	d_2	p	$x_1 - x_1^d$	$x_2 - x_2^d$	$x_3 - x_3^d$
0	0	0.0	$4.416 \cdot 10^{-3}$	$5.452 \cdot 10^{-3}$	$1.101 \cdot 10^{-2}$
6	8	0.8	$4.504 \cdot 10^{-3}$	$5.505 \cdot 10^{-3}$	$1.130 \cdot 10^{-2}$
9	10	0.8	$4.473 \cdot 10^{-3}$	$5.493 \cdot 10^{-3}$	$1.106 \cdot 10^{-2}$
12	15	0.8	$4.408 \cdot 10^{-3}$	$5.423 \cdot 10^{-3}$	$1.094 \cdot 10^{-2}$
18	20	0.6	$4.533 \cdot 10^{-3}$	$5.785 \cdot 10^{-3}$	$1.139 \cdot 10^{-2}$
25	30	0.6	$4.529 \cdot 10^{-3}$	$5.755 \cdot 10^{-3}$	$1.149 \cdot 10^{-2}$

8. Conclusions

This chapter has proposed a solution to the problem of state estimation-based control under communication delays and packet drops. The considered approach was within the frame of distributed Kalman Filtering. First, the Extended Information Filter was presented as a basic approach to nonlinear distributed filtering. The Extended Information Filter (EIF) performs fusion of the the state estimates provided by the local monitoring stations, under the assumption of Gaussian noises. The Extended Information Filter is a generalization of the Information Filter in which the local filters do not exchange raw measurements but send to an aggregation filter their local information matrices (local inverse covariance matrices or differently known as Fisher Information Matrices) and their associated local information state vectors (products of the local information matrices with the local state vectors).

To improve the estimation accuracy and convergence properties of the Extended Information Filter, the derivative-free Extended Information Filter has been introduced. The derivative-free Extended Information Filter, has the following features (i) it is not based on local linearization of the controlled system dynamics, (ii) it does not assume truncation of higher order Taylor expansion terms, (iii) it does not require the computation of Jacobian matrices. In the proposed filtering method, the system is first subject to a linearization transformation and next state estimation is performed by applying local Kalman Filters to the linearized model. The class of systems to which the derivative-free Extended Information Filter can be applied has been also defined.

Next, distributed state-estimation under communication delays and packet drops was examined. First, results on networked linear Kalman Filtering were overviewed. These results were generalized in the case of the derivative-free Extended Information Filter, where the problem of communication delays and packet drops has again the following forms: (i) there are time delays and packet drops in the transmission of information between the distributed local filters and the master filter, (ii) there are time delays and packet drops in the transmission of information from distributed sensors to each one of the local filters. In the first case, the structure and calculations of the master filter for estimating the aggregate state vector remain unchanged. In the second case, the effect of the random delays and packets drops has to be

taken into account in the redesign of the local Kalman Filters, which implies (i) a modified Riccati equation for the computation of the covariance matrix of the state vector estimation error, (ii) the use of a correction term in the update of the state vector's estimate so as to compensate for delayed measurements arriving at the local Kalman Filters.
In the simulation experiments it was shown that the aggregate state vector produced by the derivative-free Extended Information Filter can be used for sensorless control and robotic visual servoing. Visual servoing over a cameras network was considered for the nonlinear dynamic model of a planar single-link robotic manipulator. The position of the robot's end effector in the cartesian space (and equivalently the angle of the robotic link) was measured through m cameras. In turn m distributed derivative-free Kalman Filters were used to estimate the state vector of the robotic link. Next, the local state estimates were fused with the use of the standard Information Filter. Finally, the aggregate estimation of the state vector was used in a control loop which enabled the robotic link to perform trajectory tracking. It was shown that the proposed redesign of the local derivative-free Kalman filters enabled to compensate for communication delays and packet drops, thus also improving the accuracy of the presented distributed filtering approach and the robustness of the associated control loop.

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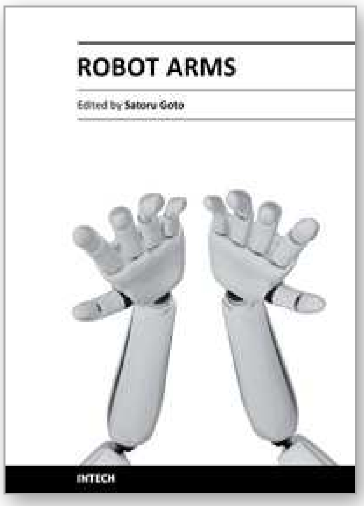

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Robot arms have been developing since 1960's, and those are widely used in industrial factories such as welding, painting, assembly, transportation, etc. Nowadays, the robot arms are indispensable for automation of factories. Moreover, applications of the robot arms are not limited to the industrial factory but expanded to living space or outer space. The robot arm is an integrated technology, and its technological elements are actuators, sensors, mechanism, control and system, etc.

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