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# Design of Controllers for Time Delay Systems: Integrating and Unstable Systems

Petr Dostál, František Gazdoš, and Vladimír Bobál  
*Faculty of Applied Informatics, Tomas Bata University in Zlín  
Nad Stráněmi 4511, 760 05 Zlín 5,  
Czech Republic*

## 1. Introduction

The presence of a time delay is a common property of many technological processes. In addition, a part of time delay systems can be unstable or have integrating properties. Typical examples of such processes are e.g. pumps, liquid storing tanks, distillation columns or some types of chemical reactors.

Plants with a time delay often cannot be controlled by usual controllers designed without consideration of the dead-time. There are various ways to control such systems. A number of methods utilise PI or PID controllers in the classical feedback closed-loop structure, e.g. (Park et al., 1998; Zhang and Xu, 1999; Wang and Cluett, 1997; Silva et al., 2005). Other methods employ ideas of the IMC (Tan et al., 2003) or robust control (Prokop and Corriou, 1997). Control results of a good quality can be achieved by modified Smith predictor methods, e.g. (Åström et al., 1994; De Paor, 1985; Liu et al., 2005; Majhi and Atherton, 1999; and Matausek and Micic, 1996).

Principles of the methods used in this work and design procedures in the 1DOF and 2DOF control system structures can be found in papers of authors of this article (Dostál et al., 2001; Dostál et al., 2002). The control system structure with two feedback controllers is considered (Dostál et al., 2007; Dostál et al., 2008). The procedure of obtaining controllers is based on the time delay first order Padé approximation and on the polynomial approach (Kučera, 1993). For tuning of the controller parameters, the pole assignment method exploiting the LQ control technique is used (Hunt et al., 1993). The resulting proper and stable controllers obtained via polynomial Diophantine equations and spectral factorization techniques ensure asymptotic tracking of step references as well as step disturbances attenuation. Structures of developed controllers together with analytically derived formulas for computation of their parameters are presented for five typical plant types of integrating and unstable time delay systems: an integrating time delay system (ITDS), an unstable first order time delay system (UFOTDS), an unstable second order time delay system (USOTDS), a stable first order plus integrating time delay system (SFOPITDS) and an unstable plus integrating time delay system (UFOPITDS). Presented simulation results document usefulness of the proposed method providing stable control responses of a good quality also for a higher ratio between the time delay and unstable time constants of the controlled system.

## 2. Approximate transfer functions

The transfer functions in the sequence ITDS, UFOTDS, USOTDS, SFOPITDS and UFOPITDS have these forms:

$$G_1(s) = \frac{K}{s} e^{-\tau_d s} \quad (1)$$

$$G_2(s) = \frac{K}{\tau s - 1} e^{-\tau_d s} \quad (2)$$

$$G_3(s) = \frac{K}{(\tau_1 s - 1)(\tau_2 s + 1)} e^{-\tau_d s} \quad (3)$$

$$G_{4,5}(s) = \frac{K}{s(\tau s \pm 1)} e^{-\tau_d s}. \quad (4)$$

Using the first order Padé approximation, the time delay term in (1) - (4) is approximated by

$$e^{-\tau_d s} \approx \frac{2 - \tau_d s}{2 + \tau_d s}. \quad (5)$$

Then, the approximate transfer functions take forms

$$G_{A1}(s) = \frac{K(2 - \tau_d s)}{s(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s} \quad (6)$$

where  $b_0 = \frac{2K}{\tau_d}$ ,  $b_1 = K$  and  $a_1 = \frac{2}{\tau_d}$  for the ITDS,

$$G_{A2}(s) = \frac{K(2 - \tau_d s)}{(\tau s - 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^2 + a_1 s + a_0} \quad (7)$$

with  $b_0 = \frac{2K}{\tau \tau_d}$ ,  $b_1 = \frac{K}{\tau}$ ,  $a_0 = -\frac{2}{\tau \tau_d}$ ,  $a_1 = \frac{2\tau - \tau_d}{\tau \tau_d}$  and  $\tau_d \neq 2\tau$  for the UFOTDS,

$$G_{A3}(s) = \frac{K(2 - \tau_d s)}{(\tau_1 s - 1)(\tau_2 s + 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^3 + a_2 s^2 + a_1 s - a_0} \quad (8)$$

where

$$b_0 = \frac{2K}{\tau_1 \tau_2 \tau_d}, \quad b_1 = \frac{K}{\tau_1 \tau_2}, \quad a_0 = \frac{2}{\tau_1 \tau_2 \tau_d}, \quad a_1 = \frac{2(\tau_1 - \tau_2) - \tau_d}{\tau_1 \tau_2 \tau_d},$$

$a_2 = \frac{2\tau_1 \tau_2 + \tau_1 \tau_d - \tau_2 \tau_d}{\tau_1 \tau_2 \tau_d}$  and  $\tau_d \neq 2\tau_1$  for the USOTDS, and,

$$G_{A4,5}(s) = \frac{K(2 - \tau_d s)}{s(\tau s \pm 1)(2 + \tau_d s)} = \frac{b_0 - b_1 s}{s^3 + a_2 s^2 + a_1 s} \quad (9)$$

where  $b_0 = \frac{2K}{\tau\tau_d}$ ,  $b_1 = \frac{K}{\tau}$ ,  $a_1 = \pm \frac{2}{\tau\tau_d}$ ,  $a_2 = \frac{2\tau \pm \tau_d}{\tau\tau_d}$  and  $\tau_d \neq 2\tau$  for the SFOPITDS and UFOPTDS, respectively.

All approximate transfer functions (6) – (9) are strictly proper transfer functions

$$G_A(s) = \frac{b(s)}{a(s)} \quad (10)$$

where  $b$  and  $a$  are coprime polynomials in  $s$  that fulfill the inequality  $\deg b < \deg a$ .

The polynomial  $a(s)$  in their denominators can be expressed as a product of the stable and unstable part

$$a(s) = a^+(s)a^-(s) \quad (11)$$

so that for ITDS, UFOTDS, USOTDS and SFOPITDS the equality

$$\deg a^+ = \deg a - 1 \quad (12)$$

is fulfilled.

### 3. Control system description

The control system with two feedback controllers is depicted in Fig. 1. In the scheme,  $w$  is the reference,  $v$  is the load disturbance,  $e$  is the tracking error,  $u_0$  is the controller output,  $y$  is the controlled output,  $u$  is the control input and  $G_A$  represents one of the approximate transfer functions (6) – (9) in the general form (10).

*Remark:* Here, the approximate transfer function  $G_A$  is used only for a controller derivation. For control simulations, the models  $G_1 - G_5$  are utilized.

Both  $w$  and  $v$  are considered to be step functions with Laplace transforms

$$W(s) = \frac{w_0}{s}, \quad V(s) = \frac{v_0}{s}. \quad (13)$$

The transfer functions of controllers are assumed as

$$Q(s) = \frac{\tilde{q}(s)}{\tilde{p}(s)}, \quad R(s) = \frac{r(s)}{\tilde{p}(s)} \quad (14)$$

where  $\tilde{q}, r$  and  $\tilde{p}$  are polynomials in  $s$ .

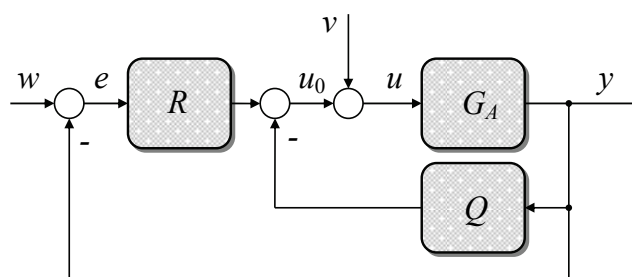


Fig. 1. The control system.

#### 4. Application of the polynomial method

The controller design described in this section follows the polynomial approach. General requirements on the control system are formulated as its internal properness and strong stability (in addition to the control system stability, also the controller stability is required), asymptotic tracking of the reference and load disturbance attenuation. The procedure to derive admissible controllers can be performed as follows:

Transforms of basic signals in the closed-loop system from Fig.1 take following forms (for simplification, the argument  $s$  is in some equations omitted)

$$Y(s) = \frac{b}{d} [rW(s) + \tilde{p}V(s)] \quad (15)$$

$$E(s) = \frac{1}{d} [(a\tilde{p} + b\tilde{q})W(s) - b\tilde{p}V(s)] \quad (16)$$

$$U(s) = \frac{a}{d} [rW(s) + \tilde{p}V(s)] \quad (17)$$

where

$$d(s) = a(s)\tilde{p}(s) + b(s)[r(s) + \tilde{q}(s)] \quad (18)$$

is the characteristic polynomial with roots as poles of the closed-loop.

Establishing the polynomial  $t$  as

$$t(s) = r(s) + \tilde{q}(s) \quad (19)$$

and substituting (19) into (18), the condition of the control system stability is ensured when polynomials  $\tilde{p}$  and  $t$  are given by a solution of the polynomial Diophantine equation

$$a(s)\tilde{p}(s) + b(s)t(s) = d(s) \quad (20)$$

with a stable polynomial  $d$  on the right side.

With regard to transforms (13), the asymptotic tracking and load disturbance attenuation are provided by divisibility of both terms  $a\tilde{p} + b\tilde{q}$  and  $\tilde{p}$  in (16) by  $s$ . This condition is fulfilled for polynomials  $\tilde{p}$  and  $\tilde{q}$  having forms

$$\tilde{p}(s) = sp(s), \quad \tilde{q}(s) = sq(s). \quad (21)$$

Subsequently, the transfer functions (14) take forms

$$Q(s) = \frac{q(s)}{p(s)}, \quad R(s) = \frac{r(s)}{sp(s)} \quad (22)$$

and, a stable polynomial  $p(s)$  in their denominators ensures the stability of controllers (the strong stability of the control system).

The control system satisfies the condition of internal properness when the transfer functions of all its components are proper. Consequently, the degrees of polynomials  $q$  and  $r$  must fulfil these inequalities

$$\deg q \leq \deg p, \quad \deg r \leq \deg p + 1. \quad (23)$$

Now, the polynomial  $t$  can be rewritten to the form

$$t(s) = r(s) + sq(s). \quad (24)$$

Taking into account solvability of (20) and conditions (23), the degrees of polynomials in (19) and (20) can be easily derived as

$$\deg t = \deg r = \deg a, \quad \deg q = \deg a - 1, \quad \deg p \geq \deg a - 1, \quad \deg d \geq 2 \deg a. \quad (25)$$

Denoting  $\deg a = n$ , polynomials  $t$ ,  $r$  and  $q$  have forms

$$t(s) = \sum_{i=0}^n t_i s^i, \quad r(s) = \sum_{i=0}^n r_i s^i, \quad q(s) = \sum_{i=1}^n q_i s^{i-1} \quad (26)$$

and, relations among their coefficients are

$$r_0 = t_0, \quad r_i + q_i = t_i \quad \text{for } i = 1, \dots, n \quad (27)$$

Since by a solution of the polynomial equation (20) only coefficients  $t_i$  can be calculated, unknown coefficients  $r_i$  and  $q_i$  can be obtained by a choice of selectable coefficients  $\gamma_i \in \langle 0, 1 \rangle$  such that

$$r_i = \gamma_i t_i, \quad q_i = (1 - \gamma_i) t_i \quad \text{for } i = 1, \dots, n. \quad (28)$$

The coefficients  $\gamma_i$  divide a weight between numerators of transfer functions  $Q$  and  $R$ .

*Remark:* If  $\gamma_i = 1$  for all  $i$ , the control system in Fig. 1 reduces to the 1DOF control configuration ( $Q = 0$ ). If  $\gamma_i = 0$  for all  $i$ , and, both reference and load disturbance are step functions, the control system corresponds to the 2DOF control configuration.

The controller parameters then result from solutions of the polynomial equation (20) and depend upon coefficients of the polynomial  $d$ . The next problem here is to find a stable polynomial  $d$  that enables to obtain acceptable stabilizing and stable controllers.

## 5. Pole assignment

The polynomial  $d$  is considered as a product of two stable polynomials  $g$  and  $m$  in the form

$$d(s) = g(s)m(s) \quad (29)$$

where the polynomial  $g$  is a monic form of the polynomial  $g'$  obtained by the spectral factorization

$$[sa(s)]^* \varphi [sa(s)] + b^*(s)b(s) = g'^*(s)g'(s) \quad (30)$$

where  $\varphi > 0$  is the weighting coefficient.

*Remark:* In the LQ control theory, the polynomial  $g'$  results from minimization of the quadratic cost function

$$J = \int_0^{\infty} \{ e^2(t) + \varphi \dot{u}^2(t) \} dt \quad (31)$$

where  $e(t)$  is the tracking error and  $\dot{u}(t)$  is the control input derivative. The second polynomial  $m$  ensuring properness of controllers is given as

$$m(s) = a^+(s) = s + \frac{2}{\tau_d} \quad (32)$$

for both ITDS and UFOTDS,

$$m(s) = a^+(s) = \left( s + \frac{2}{\tau_d} \right) \left( s + \frac{1}{\tau_2} \right) \quad (33)$$

for the USOTDS, and,

$$m(s) = \left( s + \frac{2}{\tau_d} \right) \left( s + \frac{1}{\tau} \right). \quad (34)$$

for both UFOPITDS and SFOPITDS.

The coefficients of the polynomial  $d$  include only a single selectable parameter  $\varphi$  and all other coefficients are given by parameters of polynomials  $b$  and  $a$ . Consequently, the closed loop poles location can be affected by a single selectable parameter. As known, the closed loop poles location determines both step reference and step load disturbance responses. However, with respect to the transform (13), it may be expected that weighting coefficients  $\gamma$  influence only step reference responses.

Then, the monic polynomial  $g$  and derived formulas for their parameters have forms

$$g(s) = s^3 + g_2 s^2 + g_1 s + g_0 \quad (35)$$

for both ITDS and UFOTDS, where

$$g_0 = \frac{2K}{\tau_d} \sqrt{\frac{1}{\varphi}}, \quad g_1 = \sqrt{\frac{1}{\varphi} \left( \frac{4K}{\tau_d} g_2 + K^2 \right)}, \quad g_2 = \sqrt{\frac{2}{\sqrt{\varphi}} g_1 + \frac{4}{\tau_d^2}} \quad (36)$$

for the ITDS, and,

$$g_0 = \frac{2K}{\tau \tau_d} \frac{1}{\sqrt{\varphi}}, \quad g_1 = \frac{1}{\tau \tau_d} \sqrt{4 \left( K \tau \tau_d \frac{1}{\varphi} g_2 + 1 \right) + K^2 \tau_d^2 \frac{1}{\varphi}}, \quad (37)$$

$$g_2 = \frac{1}{\tau \tau_d} \sqrt{2 \tau^2 \tau_d^2 \sqrt{\frac{1}{\varphi}} g_1 + 4 \tau^2 + \tau_d^2}$$

for the UFOTDS, and,

$$g(s) = s^4 + g_3 s^3 + g_2 s^2 + g_1 s + g_0 \quad (38)$$

for USOTDS, SFOPITDS and UFOPITDS, where

$$\begin{aligned} g_0 &= \frac{2K}{\tau_1 \tau_2 \tau_d} \frac{1}{\sqrt{\varphi}}, \quad g_1 = \sqrt{\left( \frac{4K}{\tau_1 \tau_2 \tau_d} g_2 + \frac{K^2}{\tau_1^2 \tau_2^2} \right) \frac{1}{\varphi} + \frac{4}{\tau_1^2 \tau_2^2 \tau_d^2}} \\ g_2 &= \sqrt{\frac{2}{\varphi} g_1 g_3 - \frac{4K}{\tau_1 \tau_2 \tau_d} \sqrt{\frac{1}{\varphi} + \frac{4(\tau_1^2 + \tau_2^2) + \tau_d^2}{\tau_1^2 \tau_2^2 \tau_d^2}}}, \quad g_3 = \sqrt{\frac{2}{\sqrt{\varphi}} g_2 + \frac{4}{\tau_d^2} + \frac{1}{\tau_1^2} + \frac{1}{\tau_2^2}} \end{aligned} \quad (39)$$

for the USOTDS, and,

$$\begin{aligned} g_0 &= \frac{2K}{\tau \tau_d} \frac{1}{\sqrt{\varphi}}, \quad g_1 = \sqrt{\frac{1}{\varphi} \left( \frac{4K}{\tau \tau_d} g_2 + \frac{K^2}{\tau^2} \right)}, \\ g_2 &= \sqrt{2g_1 g_3 + \frac{4}{\tau \tau_d} \left( \frac{1}{\tau \tau_d} - K \frac{1}{\sqrt{\varphi}} \right)}, \quad g_3 = \sqrt{\frac{2}{\sqrt{\varphi}} g_2 + \frac{4}{\tau_d^2} + \frac{1}{\tau^2}} \end{aligned} \quad (40)$$

for both SFOPITDS and UFOPITDS.

The transfer functions of controllers are

$$Q(s) = \frac{q_2 s + q_1}{s + p_0}, \quad R(s) = \frac{r_2 s^2 + r_1 s + r_0}{s(s + p_0)} \quad (41)$$

for both ITDS and UFOTDS, and,

$$Q(s) = \frac{q_3 s^2 + q_2 s + q_1}{s^2 + p_1 s + p_0}, \quad R(s) = \frac{r_3 s^3 + r_2 s^2 + r_1 s + r_0}{s(s^2 + p_1 s + p_0)} \quad (42)$$

for the USOTDS, SFOPITDS and UFOPITDS.

## 6. Controller parameters

For the sake of limited space, formulas derived from (20) for all considered systems together with conditions of the controllers' stability are introduced in the form of tables. Parameters  $r_i$  and  $q_i$  in (41) and (42) can then be calculated from  $t_i$  according to (28).

$p_0 = g_2 + \frac{\tau_d}{4}(2g_1 + \tau_d g_0), \quad t_0 = \frac{1}{K} g_0$ $t_1 = \frac{1}{K}(g_1 + \tau_d g_0), \quad t_2 = \frac{\tau_d}{4K}(2g_1 + \tau_d g_0)$
$p_0 > 0 \text{ for all } \tau_d$

Table 1. Controller parameters for the ITDS



$p_0 = \frac{\tau \left[ 2g_2 + \tau_d \left( g_1 + \frac{\tau_d}{2} g_0 \right) \right] + 2}{2\tau - \tau_d}$ $t_0 = \frac{\tau}{K} g_0, \quad t_1 = \frac{1}{K} \left[ p_0 + \tau (g_1 + \tau_d g_0) \right], \quad t_2 = \frac{1}{K} \left[ \tau (p_0 - g_2) - 1 \right]$
$p_0 > 0 \text{ for } \tau_d < 2\tau$

Table 2. Controller parameters for the UFOTDS

$p_0 = \frac{2g_3 + \tau_1 \left[ 2g_2 + \tau_d \left( g_1 + \frac{\tau_d}{2} g_0 \right) \right] + \frac{2}{\tau_1}}{2\tau_1 - \tau_d}, \quad p_1 = g_3 + \frac{1}{\tau_1}$ $t_0 = \frac{\tau_1}{K} g_0, \quad t_1 = \frac{1}{K} \left[ p_0 + \tau_1 (g_1 + (\tau_2 + \tau_d) g_0) \right]$ $t_2 = \frac{1}{K} \left[ \left[ \left( \frac{4\tau_1\tau_2}{\tau_d} + \tau_1 - \tau_2 \right) p_0 - \left( \frac{4\tau_2}{\tau_d} + 1 \right) \left( g_3 + \tau_1 g_2 + \frac{1}{\tau_1} \right) - \tau_1 \tau_2 g_1 \right] \right]$ $t_3 = \frac{\tau_2}{K} \left[ \tau_1 (p_0 - g_2) - g_3 - \frac{1}{\tau_1} \right]$
$p_1 > 0 \text{ for all } \tau_d, \quad p_0 > 0 \text{ for } \tau_d < 2\tau_1$

Table 3. Controller parameters for the USOTDS

$p_0 = g_2 + \frac{\tau_d}{4} (2g_1 + \tau_d g_0), \quad p_1 = g_3 + \frac{1}{\tau}$ $t_0 = \frac{1}{K} g_0, \quad t_1 = \frac{1}{K} [g_1 + (\tau + \tau_d) g_0], \quad t_2 = \frac{1}{4K} [(2\tau + \tau_d)(2g_1 + \tau_d g_0) + 2\tau \tau_d g_0]$ $t_3 = \frac{\tau \tau_d}{4K} [2g_1 + \tau_d g_0]$
$p_1, p_0 > 0 \text{ for all } \tau_d$

Table 4. Controller parameters for the SFOPITDS

$p_0 = \frac{4g_3 + (2\tau + \tau_d) \left( g_2 + \frac{\tau_d}{2} g_1 + \frac{\tau_d^2}{4} g_0 \right) + \frac{4}{\tau}}{2\tau - \tau_d}, \quad p_1 = g_3 + \frac{2}{\tau}$ $t_0 = \frac{1}{K} g_0, \quad t_1 = \frac{1}{K} [g_1 + (\tau + \tau_d) g_0],$ $t_2 = \frac{1}{K} \left[ \left( \frac{4\tau}{\tau_d} - 1 \right) p_0 - \frac{8}{\tau_d} g_3 - \left( \frac{4\tau}{\tau_d} + 1 \right) g_2 - \tau g_1 - \frac{8}{\tau \tau_d} \right], \quad t_3 = \frac{1}{K} \left[ \tau (p_0 - g_2) - 2g_3 - \frac{2}{\tau} \right]$
$p_1 > 0 \text{ for all } \tau_d, \quad p_0 > 0 \text{ for } \tau_d < 2\tau$

Table 5. Controller parameters for the UFOPITDS

## 7. Simulation results

The simulations were performed by MATLAB-Simulink tools. For all simulations, the unit step reference  $w$  was introduced at the time  $t = 0$  and the step load disturbance  $v$  after settling of the step reference responses.

### 7.1 ITDS

In the transfer function (1), let  $K = 1$ . The responses in Fig. 2 for  $\tau_d = 5$  show the effect of  $\phi$  upon the control quality. An increasing value  $\phi$  improves control stability, and, by choosing its value higher, aperiodic responses can be obtained. Simulation results shown in Fig. 3 demonstrate the influence of parameters  $\gamma$  on the control responses. Their smaller values accelerate step reference responses but they do not affect load disturbance responses. Higher values of  $\gamma$  can lead to overshoots and oscillations. The effect of parameters  $\gamma$  on the control

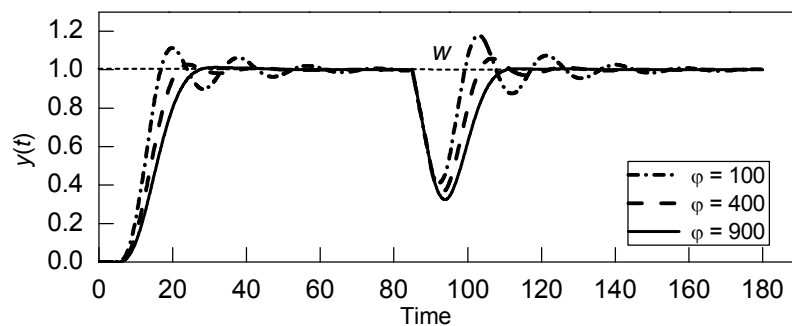


Fig. 2. ITDS: controlled output responses ( $\tau_d = 5$ ,  $v = -0.1$ ,  $\gamma_1 = \gamma_2 = 0$ )

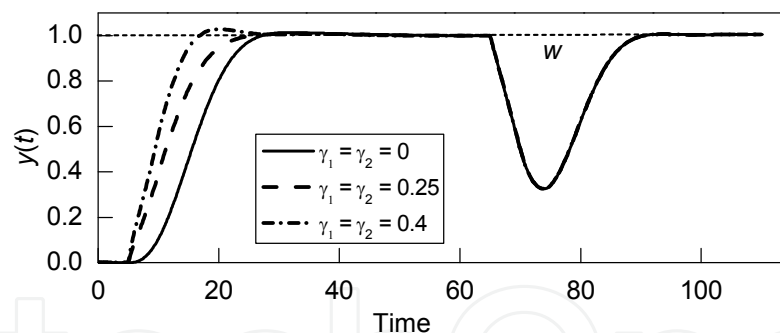


Fig. 3. ITDS: controlled output response ( $\tau_d = 5$ ,  $v = -0.1$ ,  $\phi = 900$ ).

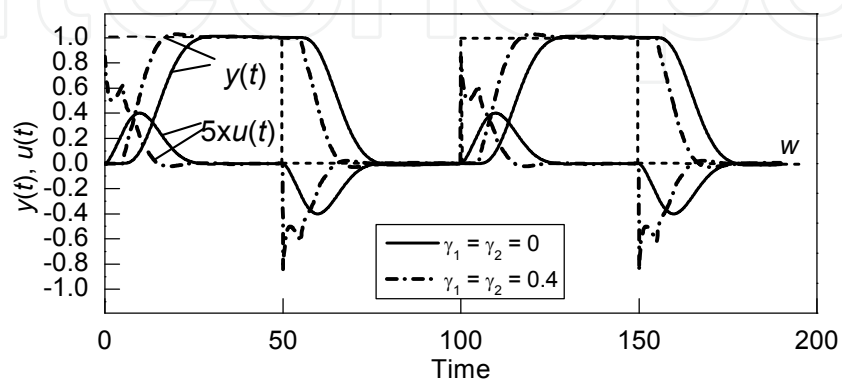


Fig. 4. ITDS: Control input and controlled output responses ( $\tau_d = 5$ ,  $\phi = 900$ )

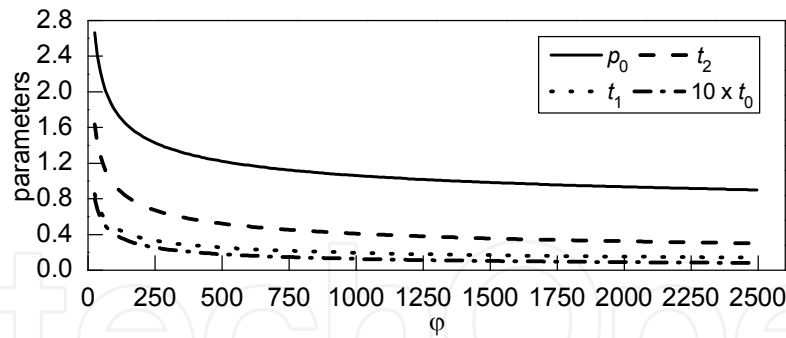


Fig. 5. ITDS: Controller parameters' dependence on  $\phi$  ( $\tau_d = 5$ )

input can be seen in Fig.4. Their higher values result in greater control inputs and their changes. This fact can be important in control of realistic processes. Dependence of the controller parameters on  $\phi$  for  $\tau_d = 5$  is shown in Fig. 5.

## 7.2 UFOTDS

In this case, the parameters in (2) have been chosen as  $K = 4$ ,  $\tau = 4$ . The effect of  $\phi$  on the control responses is similar to the ITDS, as shown in Fig. 6. The control responses for limiting values  $\gamma_1 = \gamma_2 = 1$  and  $\gamma_1 = \gamma_2 = 0$  (corresponding to the 1DOF and 2DOF structure) are in Fig. 7. The responses document unsuitability of the 1DOF structure application. The control response for  $\tau_d = 4$  is shown in Fig. 8. The presented response without any overshoots documents usefulness of the proposed method also for relatively high values of  $\tau_d$ . The responses in Fig. 9 demonstrate robustness of the proposed method against changes of  $\tau_d$ .

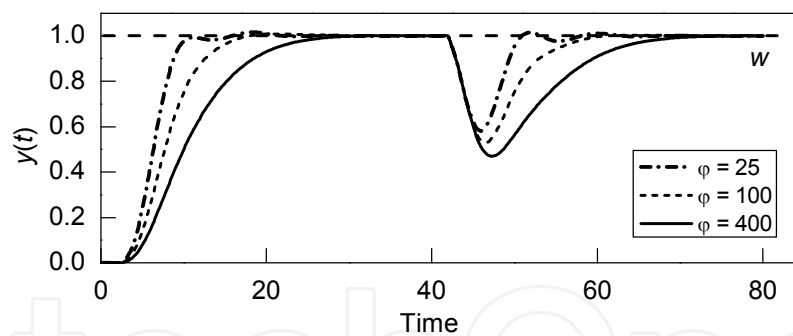


Fig. 6. UFOTDS: controlled output responses ( $\tau_d = 2$ ,  $v = -0.1$ ,  $\gamma_1 = \gamma_2 = 0$ )

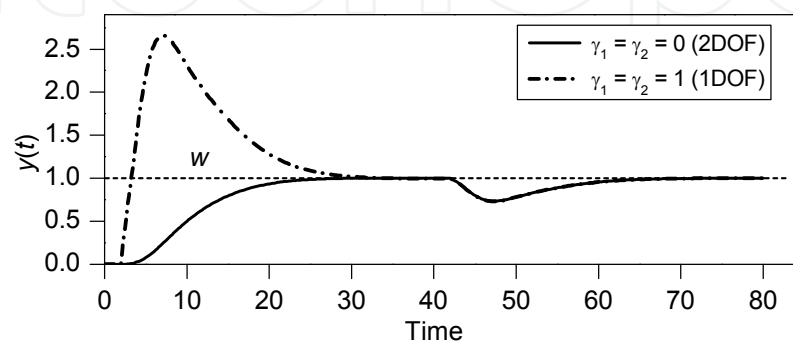


Fig. 7. UFOTDS: controlled output responses ( $\tau_d = 2$ ,  $v = -0.05$ ,  $\phi = 400$ )

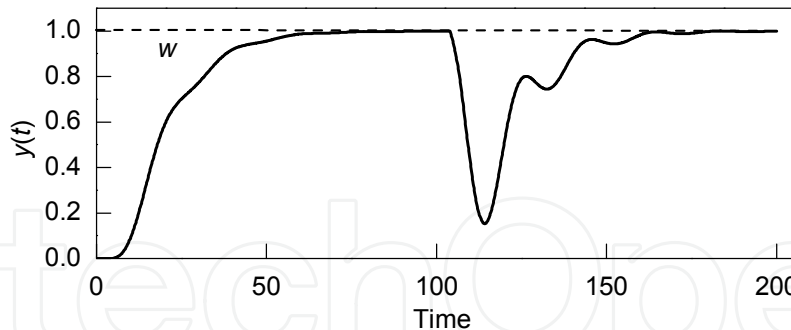


Fig. 8. UFOTDS: controlled output response ( $\tau_d = 4$ ,  $v = -0.05$ ,  $\phi = 2500$ ,  $\gamma_1 = \gamma_2 = 0$ )

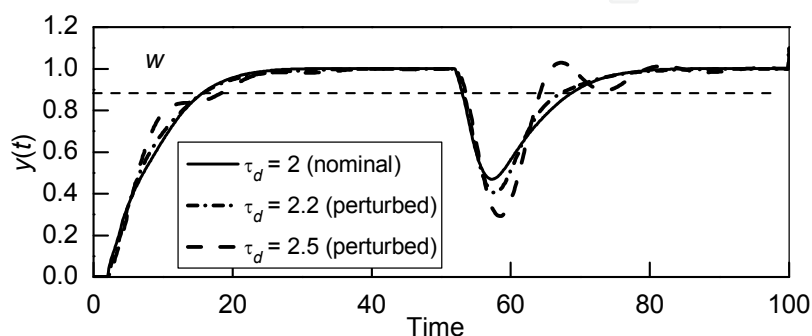


Fig. 9. Robustness against a change of  $\tau_d$  ( $v = -0.1$ ,  $\phi = 400$ ,  $\gamma_1 = \gamma_2 = 0$ )

The controller parameters were computed for a nominal model with  $\tau_d = 2$  and subsequently used for perturbed models with the +10% and +25% estimation error in the  $\tau_d$ .

### 7.3 USOTDS

In this case, the parameters in (3) were selected to be  $K = 1$ ,  $\tau_1 = 4$ ,  $\tau_2 = 2$ . Analogous to controlling the UFOTDS, the responses in Fig. 10 prove applicability of the proposed method also for an USOTDS with a relatively high ratio between the time delay and an unstable time constant ( $\tau_d / \tau_1 = 1$ ). The responses in Fig. 11 demonstrate the possibility of extensive control acceleration, and, also high sensitivity of the control responses to the selection of parameters  $\gamma$ .

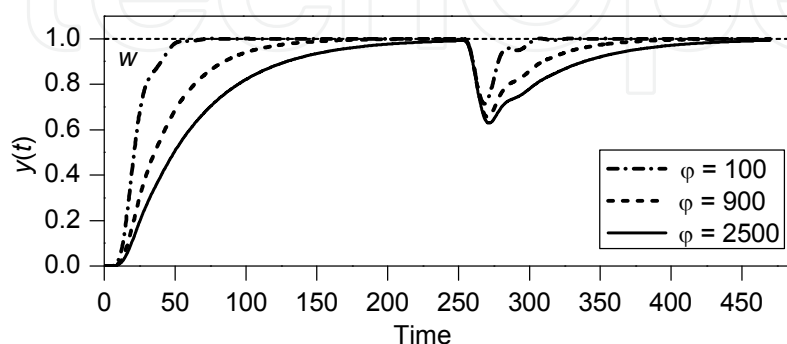


Fig. 10. USOTDS: controlled output responses ( $\tau_d = 4$ ,  $v = -0.05$ ,  $\gamma_1 = \gamma_2 = \gamma_3 = 0$ )

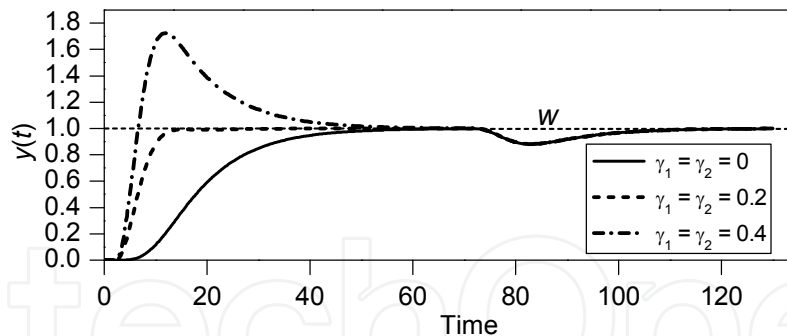


Fig. 11. USOTDS: controlled output responses ( $\tau_d = 2$ ,  $v = -0.05$ ,  $\phi = 100$ ,  $\gamma_3 = 0$ )

#### 7.4 SFOPITDS

For this model, the parameters in (4) have been chosen as  $K = 1$ ,  $\tau = 4$ ,  $\tau_d = 4$ . A suitable selection of parameters  $\phi$  and  $\gamma$  provides control responses of a good quality, as illustrated in Figs. 12 and 13.

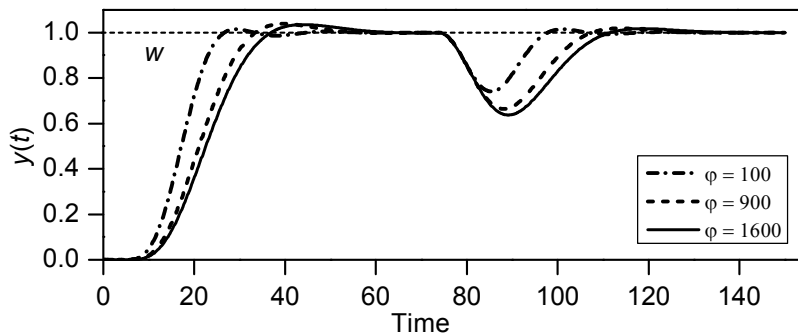


Fig. 12. SFOPITDS: controlled output responses ( $\tau_d = 4$ ,  $v = -0.05$ ,  $\gamma_1 = \gamma_2 = \gamma_3 = 0$ )

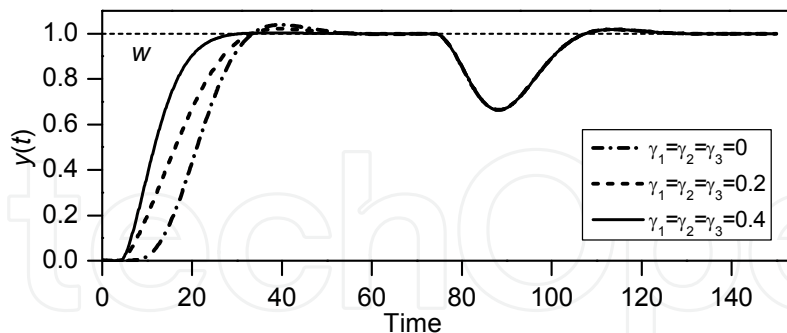


Fig. 13. SFOPITDS: controlled output responses ( $\tau_d = 4$ ,  $v = -0.05$ ,  $\phi = 900$ ).

#### 7.5 UFOPITDS

Here, the model parameters in (4) have been chosen the same as for the SFOPITDS. With regard to the presence of both integrating and unstable parts, the UFOPITDS belongs to hardly controllable systems. However, the control responses in Fig. 14 document usefulness of the proposed method also for such systems. Obviously, for higher values  $\tau_d$  also higher values of  $\phi$  have to be chosen. Moreover, for this system, only the 2DOF structure with zero parameters  $\gamma$  should be used as follows from Fig. 15.

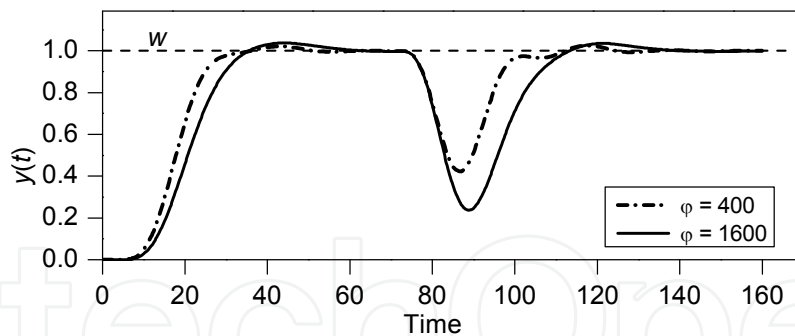


Fig. 14. UFOPITDS: controlled output responses ( $\tau_d = 3, v = -0.025, \gamma_1 = \gamma_2 = \gamma_3 = 0$ )

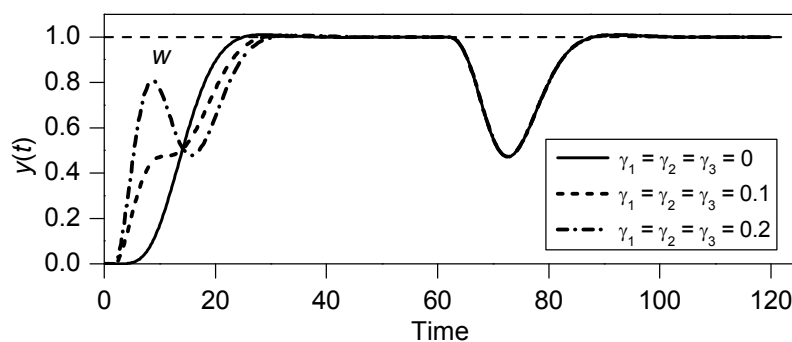


Fig. 15. UFOPITDS: controlled output responses ( $\tau_d = 2, v = -0.05, \phi = 100$ )

## 8. Conclusions

The problem of control design for integrating and unstable time delay systems has been solved and analysed. The proposed method is based on the Padé time delay approximation. The controller design uses the polynomial synthesis and results of the LQ control theory. The presented procedure provides satisfactory control responses for the tracking of a step reference as well as for the step load disturbance attenuation. The procedure enables tuning of the controller parameters by two types of selectable parameters. Using derived formulas, the controller parameters can be automatically computed. As a consequence, the method could also be used for adaptive control.

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Time delay is very often encountered in various technical systems, such as electric, pneumatic and hydraulic networks, chemical processes, long transmission lines, robotics, etc. The existence of pure time lag, regardless if it is present in the control or/and the state, may cause undesirable system transient response, or even instability. Consequently, the problem of controllability, observability, robustness, optimization, adaptive control, pole placement and particularly stability and robustness stabilization for this class of systems, has been one of the main interests for many scientists and researchers during the last five decades.

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Phone: +86-21-62489820  
Fax: +86-21-62489821



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