We are IntechOpen, the world’s leading publisher of Open Access books
Built by scientists, for scientists

3,800 Open access books available
116,000 International authors and editors
120M Downloads

154 Countries delivered to
TOP 1% Our authors are among the most cited scientists
12.2% Contributors from top 500 universities

WEB OF SCIENCE™
Selection of our books indexed in the Book Citation Index in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com
1. Introduction

Water resources systems analysis is the science of developing and applying mathematical operations research methodologies to water resources systems problems comprised of reservoirs, rivers, watersheds, groundwater, distribution systems, and others, as standalone or integrated systems, for single or multiobjective problems, deterministic or stochastic. The scientific and practical challenge in dealing quantitatively with water resources systems analysis problems is in taking into consideration from a systems perspective, social, economic, environmental, and technical dimensions, and integrating them into a single framework for trading-off in time and in space competitive objectives. Inherently, such problems involve modelling of water quantity and quality for surface water, groundwater, water distribution systems, reservoirs, rivers, lakes, and other systems as stand alone or combined systems. Numerous models for water resources systems analysis have been proposed during the past four decades. A possible classification for those is into: (1) methods based on decomposition in which an "inner" linear/quadratic problem is solved for a fixed low-dimension decision variables set, while that set is altered at an "outer" problem using a gradient or a sub-gradient technique (e.g., Alperovits and Shamir, 1977; Quindry et al. 1979, 1981; Kessler and Shamir, 1989, 1991; Eiger et al., 1994; Ostfeld and Shamir, 1996), (2) methods which utilize a straightforward non-linear programming formulation (e.g., Watanatada, 1973; Shamir, 1974; Karatzas and Finder, 1996), (3) methods based on linking a simulation program with a general nonlinear optimization code (e.g., Ormsbee and Contractor, 1981; Lansey and Mays, 1989), and (4) more recently, methods which employ evolutionary techniques such as genetic algorithms (e.g., Simpson et. al, 1994; Savic and Walters, 1997; Espinoza et al., 2005; Espinoza and Minsker, 2006), Cross Entropy (e.g., Perelman and Ostfeld, 2008), or the shuffled frog leaping algorithm (e.g., Eusuff and Lansey, 2003). Among the above classification ant colony optimization (ACO) belongs to category (4) of evolutionary techniques. Although some studies have already been conducted (e.g., Maier et al., 2003, Ostfeld and Tubaltzev, 2008; Christodoulou and Ellinas, 2010) in which ant colony optimization was utilized, the employment of ACO in water resources systems studies is still in its infancy.

This book chapter reviews the current literature of ACO for water resources systems analysis, and suggests future directions and challenges for using ACO for solving water resources systems problems [parts of this Chapter are based on Ostfeld and Tubaltzev (2008), with permission from the American Society of Civil Engineers (ASCE)].
2. Ant Colony Optimization

Proposed by Dorigo (1992), Ant Colony Optimization (ACO) is an evolutionary stochastic combinatorial computational discipline inspired by the behaviour of ant colonies. One of the problems studied by ethologists is to understand how ants which are almost completely blind could manage to establish shortest paths from their nest to their feeding sources and back. It was found that ants communicate information by leaving pheromone tracks. A moving ant leaves, in varying quantities, some pheromone on the ground to mark its way. While an isolated ant moves essentially at random, an ant encountering a previously laid trail is able to detect it and decide with high probability to follow it, thus reinforcing the track with its own pheromone. The collective behavior that emerges is thus a positive feedback: where the more the ants following a track, the more attractive that track becomes for being followed; thus the probability with which an ant chooses a path increases with the number of ants that previously chose the same path. This elementary behavior inspired the development of ACO (Dorigo, 1992).

ACO has been already successfully applied to a number of NP hard combinatorial optimization problems such as the traveling salesman problem (TSP) or the quadratic assignment problem (QAP), but still to only a limited number of studies in water resources systems analysis.

Consider a colony of ants moving on a graph $G(N, E)$ where $N$ is the set of nodes (decision points) $i = 1, \ldots, N$ and $E$ is the set of edges (links) $e = 1, \ldots, E$, the basic scheme of ACO (following Dorigo et al., 1996), involves the following steps:

1. The probability of the $k$-th ant situated at node $i$ at stage $t$, to choose an outgoing edge $e$ is:

$$
P_{e,i}^k(t) = \frac{[\tau_e(t)]^{\alpha} [\eta_e]^{\beta}}{\sum_{e \in I^+(i)} [\tau_e(t)]^{\alpha} [\eta_e]^{\beta}}
$$

where:

- $P_{e,i}^k(t)$ = the probability of the $k$-th ant at node $i$ at stage $t$, to choose edge $e$;
- $\tau_e(t)$ = the pheromone intensity (quantity per unit of length) present on edge $e$ at stage $t$;
- $I^+(i)$ = the set of outgoing edges (i.e., all the outgoing allowable directions) from node $i$ at stage $t$;
- $\eta_e$, $\alpha$, and $\beta$ = parameters ($\eta_e$ = visibility, $\alpha$, $\beta$ = parameters that control the relative importance of the pheromone amount present on edge $e$ at stage $t$, see Dorigo et al., 1996).

2. The pheromone intensity at $\tau_e(t + 1)$ is updated using (2) – (4):

$$
\tau_e(t + 1) = \rho \tau_e(t) + \Delta \tau_e(t + 1)
$$

$$
\Delta \tau_e(t + 1) = \sum_{k=1}^{A} \Delta \tau_{e}^k(t + 1)
$$

$$
\Delta \tau_{e}^k(t + 1) = \begin{cases} 
R \frac{R}{C_k(t)} & \text{if the } k\text{-th ant used edge } e \text{ at stage } t \\
0 & \text{otherwise}
\end{cases}
$$

where:

- $\rho$ = a coefficient less than one representing a pheromone evaporation intensity between consecutive stages; $A$ = total number of ants; $R$ = a pheromone reward constant;
and \( C_k(t) \) = the solution cost generated by ant \( k \) during stage \( t \) [e.g., the length of a complete \( k \)-th ant tour at the traveling salesman problem (TSP) at stage \( t \)].

3. Generation of one solution [i.e., \( C_k(t) \)] by the \( k \)-th ant at stage \( t \) is termed an iteration while the completion of all ants' solutions at stage \( t \) denotes a cycle. Ants thus complete iterations by generating solutions, and once all ants have produced solutions, a cycle (i.e., a stage) has been completed. The algorithm parameters are: \( \alpha, \beta, \eta_e, \rho, R \); the number of ants \( A \), and a penalty induced for non-feasible solutions.

A pseudo-code for an ant colony optimization algorithm as described above, can take the following form:

**Initialization**
- Set: \( t = 0; \) \( \tau_e(0), \Delta \tau_e(0) = 0 \) \( \forall e \in E \); distribute the \( A \) ants (indexed \( k \)) on the \( N \) nodes (decision points).
- Set the ant colony parameters: \( \alpha, \beta, \eta_e, \rho, \) and \( R \).
- Compute \( P^k_{e,i}(0) \) \( \forall e \in E \) using (1).

**Main scheme**

**REPEAT**
- For the \( k \)-th ant situated at node \( i \) generate a solution by performing a random walk on \( G(N, E) \) using \( P^k_{e,i}(t) \) (e.g., for the TSP visit all towns exactly once).
- Compute \( C_k(t) \) - the solution cost produced by the \( k \)-th ant.

**Until** \( A \) (total number of ants)
- Update the best cost solution found.
- Compute \( \tau_e(t + 1) \) \( \forall e \in E \) using (2) – (4).
- Compute \( P^k_{e,i}(t + 1) \) \( \forall e \in E \) using (1).
- Set: \( t \leftarrow t + 1 \)

UNTIL \( T_{\text{max}} \), where: \( T_{\text{max}} \) = maximum number of cycles (stages); or until no improvement of the best solution is encountered at some consecutive stages.

3. Literature review

Ant colony optimization is a new evolutionary computational discipline to the water resources systems community. Only few models were developed thus far. Attempts were made to employ ant colony for reservoir operation, groundwater long term monitoring, for some general water resources problems, and in particular for water distribution systems design and operation. This section incorporates a brief description of these efforts.

3.1 Reservoirs optimal operation

Kumar and Reddy (2006) proposed an Ant Colony Optimization algorithm for a multi-purpose reservoir system. To tailor the ACO algorithm for their problem, a finite time series of inflows, classification of the reservoir volume into several class intervals, and defining the reservoir releases for each time period with respect to a predefined optimality criterion,
were established. The ACO algorithm was compared to a real coded Genetic Algorithm (GA). It was shown that the ACO algorithm performed better than the GA. Moeini and Afshar (2009) used three max-min ant system formulations for optimal operation of reservoirs using two sets of decision variables – storage and releases, and three graph form representations. The max-min ant system results were compared to each other and to two traditional heuristic evolutionary algorithms: genetic algorithms (GA), and honey bee mating optimization (HBMO). It was shown that the max-min ant system formulation was successful in solving the problem of optimal operation of reservoirs with the releases settings been better than the others. Dariane and Moradi (2009) used ant colony optimization for continuous domains (ACOR) to solve the optimal releases problem of reservoirs. The authors decreased substantially the computational effort required to run an ant colony based optimization problem, and compared their model to a genetic algorithm formulation.

3.2 Groundwater long term (LTM) monitoring

Li et al. (2004) developed a hybrid ant colony genetic algorithm model for groundwater long term monitoring to maximize sampling cost-effectiveness. Groundwater long term monitoring is required to evaluate the in-situ conditions of remedial system performances and for controlling post closure groundwater sites. Their formulation optimizes the groundwater-monitoring network (i.e., location and sampling schedules) as groundwater monitoring resources are expensive and thus limited. Li and Chan Hilton (2005, 2006a, 2006b) extended Li and Chan Hilton (2004) by formulating the LTM optimization for minimizing the number of monitoring wells with constraints on estimation errors and data quality; and for reduction of a monitoring effort plan while minimizing distortions to the information received by the original monitoring set-up. Global optima or near-optimal solutions were received.

3.3 Water resources applications

Ali et al. (2009) used ant colony optimization to accelerate convergence of the differential evolution (DE) technique. Their methodology, entitled ant colony differential evolution (ACDE), initializes the ant population using based learning techniques, utilizes a random localization methodology, and simulates the movement of ants to refine the best solution found in each iteration. The ACDE was applied to different test problems including a simplified water resources system. Abbaspour et al. (2001) utilized ant colony optimization for calibrating an unsaturated soil hydraulic model. The use of ant colony replaced the traditional inverse modelling approach and was found to be successful in overcoming previous parameterization related optimization problems. Li et al. (2006) developed a hybrid ant colony-simulated annealing method for groundwater parameter estimation. The inverse problem of parameter identification was formulated as an optimization problem. Transmissivity and storage coefficients for a two-dimensional unsteady state groundwater flow model were calibrated with the proposed technique.

3.4 Water distribution systems

The first to introduce ant colony optimization for water distribution systems management were Maier et. al. (2003). Maier et al. applied a traditional ant colony settings for optimizing two benchmark gravitational one loading conditions water distribution systems. Zeccchin et
al. (2005) studied parameterization of ant colony optimization for water distribution systems and suggested guidelines for selecting them. Christodoulou and Ellinas (2010) proposed an ant colony optimization algorithm for efficient routing of piping networks for improving its efficiency and resiliency. López-Ibáñez et al. (2008) used ant colony for optimizing the operation of pumping in water distribution systems. Ostfeld A. and Tubaltzev A. (2008) generalized the studies of Maier et al. (2003) and López-Ibáñez et al. (2008) for mutually optimizing the design and operation of water distribution systems for extended loading conditions, pumps, and tanks. The algorithm of Ostfeld A. and Tubaltzev A. (2008) is based on Dorigo et al. (1996) and Maier et al. (2003), and is comprised of the following stages:

1. **Representation:** the least cost design problem is discretized and symbolized in the form of a graph with the links representing decision variable values, and the nodes – decision points.
2. **Initialization:** a colony of A ants "starts walking" from node START (see Fig. 1) to node END with each ant having an equal probability to choose a specific link at each node (i.e., each link has an equal initial pheromone intensity of 1 unit). Each ant entire trail (i.e., from START to END) comprises one possible design solution. At the end of this stage a set of random design solutions (i.e., random routes) equal to the number of ants is generated. (2a) Each solution is evaluated using EPANET (USEPA, 2002) with a penalty induced to non-feasible solutions [i.e., solutions violating minimum allowable pressure constraints at consumer nodes are penalized linearly as a function of the pressure head deficiency at the nodes (following Maier et al., 2003)]. (2b) A set (denoted Δ) of the least cost solutions are selected for pheromone updating (e.g., the best twenty ant solutions out of an initial set A of thousand).
3. **Pheromone updating:** each of the links participating at the i-th best solution (i ∈ Δ) is added a pheromone amount equal to: Cost\textsubscript{max} / Cost\textsubscript{i}, where Cost\textsubscript{max} is the highest cost among the best set (i.e., Δ) of ants, and Cost\textsubscript{i} is the solution cost of the current i-th best solution. Using this mechanism links which participated at lower solutions will receive a higher pheromone quantity (i.e., their likelihood to be chosen at subsequent iterations will increase).
4. **Probability updating:** update the links outgoing probabilities out of node j: p_{i} = \frac{\text{ph}_{i}}{\sum_{i=1}^{N_{j}} \text{ph}_{i}} \text{ where: } p_{i} = \text{the probability of choosing link } i; N_{j} = \text{number of links out of node } j; \text{ and } \text{ph}_{i} = \text{pheromone amount on link } i.
5. **Iterate:** go back to stage (2b) with a fraction γ of the initial number of ants A (i.e., γA), while keeping the best solution (i.e., Elitism), if a predefined number of iterations has not yet attained.

A complete formal description of the suggested algorithm is given in Fig. 2. It incorporates the following steps:

**Initialization:**
1. The ant colony iteration counter t is set to one.
2. The pheromone intensity τ\textsubscript{e}(1) present on each edge e ∈ {1, 2,...,E} at t = 1 is set to one.
Fig. 1. Water distribution systems representation for Ant Colony Optimization (Ostfeld and Tubaltzev, 2008, with permission from the American Society of Civil Engineers (ASCE))

Legend:
- . . . . . : an ant trial;
- O: decision junction;
- $P_i$: pumping station $i$ ($i = 1 \ldots z$);
- $z$: total number of pumping stations);
- $L_j$: loading $j$ ($j = 1 \ldots nlo$, where:
  - nlo: total number of loading conditions);
- $d_i$: candidate pipe diameter $i$ ($i = 1 \ldots D$, where: $D$: total number of candidate pipe diameters);
- $F_{\max}$: maximum power of pumping station;
- $r_{\max}$: relative speed number maximum value;
- $\sigma$: discretized stages for $r_{\max}$. 
3. The ant colony parameters are defined: $t_{\text{max}}$ = total number of ant colony iterations; $\alpha$ = a parameter controlling the relative importance of pheromone (set to one); $\beta$ = a parameter controlling the relative importance of local heuristics (set to zero); $\rho$ = a parameter of pheromone persistence (set to one); $A$ = initial number of ants; $\gamma$ = a fraction of the initial number of ants used for $t > 1$ (i.e., use of $\gamma A$ ants for $t > 1$); $\Delta$ = number of best ants used for pheromone updating; $n_{\text{max}}$ = upper bound of the relative pumping stations speed number; $\sigma$ = discretized $n_{\text{max}}$ number (i.e., the discretization resolution of $n_{\text{max}}$); and $PC$ = linear penalty cost coefficient.

4. The initial number of ants $A$ are placed at node START (see Fig. 1).

5. The probability $P_{e,i}(t)$ to select the outgoing edge $e$ at node $i$ at iteration $t$ ($t = 1$) is computed:

$$P_{e,i}(t) = \frac{[\tau_{e}(t)]^\alpha [\eta_e]^{1-\alpha}}{\sum_{j=1}^{n_{\text{max}}} [\tau_{j}(t)]^{\alpha} [\eta_j]^{\beta}} \quad \forall e \in \{1, 2, \ldots, E\}$$

(5)

where: $i^+$ = the set of outgoing edges out of node $i$; $\eta_e$ = visibility of edge $e$ (not used as $\beta = 0$).

6. The ant’s trials counter $k$ is set to 1

7. The solution of the first ant ($k = 1$) trial $\phi_k(1) [P_{e,i}(1)]$ is generated through a random walk to node END (see Fig. 1).

8. The trial solution cost of the first ant ($k = 1$) $C_{1}$ is computed; Stages (9) and (10) guarantee that all ants will perform their trails. The initialization stage concludes with keeping the best $\Delta$ solutions out of the initial number of ants $A$.

Main scheme:

11. The pheromone intensity $\tau_{e}(t + 1)$ present on edge $e$ at iteration $t + 1$ is computed using Eqs. (2) and (3):

$$\tau_{e}(t + 1) = \rho \tau_{e}(t) + \sum_{k=1}^{\Delta} \Delta \tau_{e,k}^k (t + 1) \quad \forall e \in \{1, 2, \ldots, E\}$$

(6)

$$\Delta \tau_{e,k}^k (t + 1) = \begin{cases} \frac{C_{\text{max},\Lambda}(t)}{C_k(t) [\phi_k(t)]} & \text{if the k-th ant } (k \in \Delta) \text{ used edge e at iteration } t + 1 \\ 0 & \text{otherwise} \end{cases}$$

(7)

where: $\Delta \tau_{e,k}^k (t + 1)$ = pheromone intensity addition by the k-th ant on edge $e$ at iteration $t + 1$; and $C_{\text{max},\Lambda}(t)$ = the maximum cost solution out of the best $\Delta$ ants at iteration $t$. At stage (12) the probability $P_{e,i}(t + 1)$ to choose the outgoing edge $e$ at node $i$ at iteration $t + 1$ is computed.

At stage (13) $\gamma A$ ants are placed at node START (see Fig. 1). Stages (14) to (18) correspond to (6) to (10); at stage (19) Elitism is invoked (i.e., keeping unchanged the best solution obtained at all iterations); stages (20) and (21) guarantee that all iterations are performed. The algorithm finishes off with the best ant colony solution obtained $\phi (t_{\text{max}})$, and its corresponding cost $C^\star (t_{\text{max}})$. 

www.intechopen.com
Initial stage
(1) Set: \( t = 1 \)
(2) Set: \( \tau_e(1) = 1 \ \forall \ e \in \{1, 2, \ldots, E\} \)
(3) Set the algorithm parameters: \( t_{\text{max}}, \alpha = 1, \beta = 0, \rho = 1, A, \gamma, \Delta, n_{\text{max}}, \sigma, \text{PC} \)
(4) Locate A ants: \([1 \ldots k \ldots A]\) at node START (see Fig. 1)
(5) Compute: \( P_{e,i}(1) = \sum_{j=1}^{E} \left[ \frac{\tau_e(1)^{\alpha} \eta_e}{\sum_{j=1}^{E} \tau_j(t)^{\alpha} \eta_j} \right]^{\beta} \forall e \in \{1, 2, \ldots, E\} \)
(6) Set: \( k = 1 \)

(7) Generate \( \phi_k(1)[P_{e,i}(1)] \) through a random walk to node END (see Fig. 1)
(8) Compute \( C_k(1) \)
(9) Check if \( k = A \)
(10) NO – Set: \( k \leftarrow k + 1 \) and go to step (7)
YES – save the least cost set \( \delta \) (i.e. the best \( \Delta \)) solutions out of \( A \)

The ant colony main algorithm
(11) Compute: \( \tau_e(t+1) = \rho \tau_e(t) + \sum_{k=1}^{\Delta} \Delta \tau_e^k(t+1) \forall e \in \{1, 2, \ldots, E\} \)
\[ \Delta \tau_e^k(t+1) = \begin{cases} \frac{C_{\text{max}}-C_k(t)}{C_k(t)} \phi_k(t) \tau_e(t+1) & \text{if the k-th ant (k} \in \Delta \text{) used edge e at iteration t} \\ 0 & \text{otherwise} \end{cases} \]
(12) Compute: \( P_{e,i}(t+1) = \sum_{j=1}^{E} \left[ \frac{\tau_e(t+1)^{\alpha} \eta_e}{\sum_{j=1}^{E} \tau_j(t+1)^{\alpha} \eta_j} \right]^{\beta} \forall e \in \{1, 2, \ldots, E\} \)
(13) Locate \( \gamma A \) ants: \([1 \ldots k \ldots \gamma A]\) at node START (see Fig. 1)
(14) Set: \( k = 1 \)

(15) Generate \( \phi_k(t+1)[P_{e,i}(t+1)] \) through a random walk to node END (see Fig. 1)
(16) Compute \( C_k(t+1) \)
(17) Check if \( k = \gamma A \)
(18) NO – Set: \( k \leftarrow k + 1 \) and go to step (15)
YES – save the least cost set \( \delta \) (i.e. the best \( \Delta \)) solutions out of \( \gamma A \)

(19) Elitism: if \( C^*(t+1) > C^*(t) \) then: \( \phi^*(t+1) \leftarrow \phi^*(t) \)
(20) Check if \( t = t_{\text{max}} \)
(21) NO – Set: \( t \leftarrow t + 1 \) and go to step (11)
YES – save the least cost ant colony solution \( \phi^*(t_{\text{max}}) \)

Fig. 2. Ant Colony Optimization flow chart (Ostfeld and Tubaltzev, 2008, with permission from the American Society of Civil Engineers (ASCE))
4. Future research

Ant colony optimization for water resources systems analysis is in its early stages of exploitation as depicted from the literature review described above. Applications to optimal reservoir operations, long term monitoring, some general water resources problems, and water distribution systems design and operation were developed. Still, most of the water resources systems community has not yet utilized the potential of using ant colony optimization, as occurred in other research disciplines such as structural engineering. Research is thus almost completely open for developing and applying ant colony optimization algorithms for water resources systems problems and analysis.

The challenges of using ant colony optimization in water resources systems analysis vary with the specific theme and objective of interest. Yet, commonly to water resources systems analysis is the inherent probabilistic complex nature of the physical systems such as groundwater, watersheds, distribution systems, and others. Those yield non-linearity and non-smoothness in describing the systems physical behavior. As a result, almost any water resources systems model which obviously needs to capture its physics as model constraints, is highly complex. Traditional non-linear algorithms such as gradient type algorithms are thus very much limited.

As every problem has its unique structure and tradeoff among its decision variables and formulations, the challenge of developing ant colony algorithms for water resources systems analysis is mainly in tailoring the specific problems characteristics with an ant colony formulation. This requires the modeler to explore different avenues of formulations such that the resulted model will be computationally feasible. This is a tremendous challenging task which captures most of the innovativeness aspects of new modeling and application.

Below are the major fields of water resources systems analysis for which new ant colony optimization models could be developed and applied:

**Climate change**: climate variability and change, adaptive management, decision making

**Groundwater**: parameter estimation, operation, contamination, remediation

**Reservoirs**: operation, design, inclusion of water quality considerations

**Water distribution systems**: network models – optimization, calibration and verification, development and application; network hydraulics – steady state, transients; leakage – leak detection, leak management, field studies; water use – monitoring, estimation and simulation, end users; field work – tracer studies, pressure tests, case studies; contaminant intrusion and water security – detection, source identification, response; network vulnerability – security assessments, network reliability, disaster response, emerging issues; network water quality – real-time monitoring and modeling, dose exposure, mixing and dispersion, storage tanks, asset management, Maintenance, system expansion and rehabilitation; sustainable water distribution systems – design and operation, water reuse and supply, dual distribution systems

**Water economics**: water demand – household, commercial; water supply – cost of production, efficiency studies, technological change, industry studies; valuing water services – ecological services, sanitation navigation, recreation, irrigation, industrial.

**Water policy**: transboundary, drought, flood, navigation, recreation, irrigation, industrial, climate, energy

**Watersheds and river basins**: watershed management, water and energy systems, application of hydrologic predictions and forecasts, best management practices, stormwater
5. References


Li S., Liu Y., and Yu H. (2006). "Parameter estimation approach in groundwater hydrology using hybrid ant colony system." In Computational Intelligence and Bioinformatics,
Ant Colony Optimization for Water Resources Systems Analysis – Review and Challenges


Ants communicate information by leaving pheromone tracks. A moving ant leaves, in varying quantities, some pheromone on the ground to mark its way. While an isolated ant moves essentially at random, an ant encountering a previously laid trail is able to detect it and decide with high probability to follow it, thus reinforcing the track with its own pheromone. The collective behavior that emerges is thus a positive feedback: where the more the ants following a track, the more attractive that track becomes for being followed; thus the probability with which an ant chooses a path increases with the number of ants that previously chose the same path. This elementary ant's behavior inspired the development of ant colony optimization by Marco Dorigo in 1992, constructing a meta-heuristic stochastic combinatorial computational methodology belonging to a family of related meta-heuristic methods such as simulated annealing, Tabu search and genetic algorithms. This book covers in twenty chapters state of the art methods and applications of utilizing ant colony optimization algorithms. New methods and theory such as multi colony ant algorithm based upon a new pheromone arithmetic crossover and a repulsive operator, new findings on ant colony convergence, and a diversity of engineering and science applications from transportation, water resources, electrical and computer science disciplines are presented.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:
