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Robust Sampled-Data Control Design of Uncertain Fuzzy Systems with Discrete and Distributed Delays

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1. Introduction

Nonlinear time-delay systems appear in many engineering systems and system formulations such as transportation systems, networked control systems, telecommunication systems, chemical processing systems, and power systems. Hence, it is important to analyze and synthesize such time-delay systems. Considerable research on nonlinear time-delay systems has been made via fuzzy system approach in (2), (6), (9), (12), (13) where stability conditions of fuzzy systems with discrete delays have been given in terms of Linear Matrix Inequalities (LMIs). Takagi-Sugeno fuzzy systems, described by a set of if-then rules which gives local linear models of an underlying system, represent a wide class of nonlinear systems. In the last two decade, Takagi-Šugeno fuzzy system has been extensively used for nonlinear control systems since it can universally approximate or exactly describe general nonlinear systems((8)). Theory has been extended to fuzzy systems with distributed delays in (7), (11), (15). Those results are based on continuous-time delay systems. From a practical point of view, sampled-data control is of importance. However, only a few results on sampled-data control for fuzzy system with discrete delays have been given in the literature ((1), (5), (14), (16)). Sampled-data controller design has been made for fuzzy systems with distributed delays in (3) and (4). To the best of our knowledge, no result for fuzzy sampled-data control systems with neutral and distributed delays has appeared yet.

In this paper, we propose a design method for robust sampled-data control of uncertain fuzzy systems with discrete, neutral and distributed delays. A zero-order sampled-data control can be regarded as a delayed control. Hence, a time-varying delay system approach is taken to design a sampled-data controller. We first obtain a stability condition by introducing an appropriate Lyapunov-Krasovskii functional with free weighting matrices, which reduce the conservatism in our stability condition. Then, based on such an LMI condition, we propose a robust sampled-data control design method of fuzzy uncertain systems with discrete, neutral and distributed delays. We also propose a sampled-data observer design method of fuzzy time-delay systems. A similar approach is taken for analysis of a sampled-data observer, and a condition for an existence of an observer is given by another LMI, which is a dual result of stabilizing controller. Finally, we give some illustrative examples to show our design procedures for sampled-data controller and observer.
2. Fuzzy time-delay systems

In this section, we introduce Takagi-Sugeno fuzzy systems with discrete, neutral and distributed delays. Consider the Takagi-Sugeno fuzzy time-delay model, described by the following IF-THEN rule:

\[
\text{IF } \xi_i(t) = M_{i1} \text{ and } \ldots \text{ and } \xi_p(t) = M_{ip}, \\
\text{THEN } \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\gamma(t)) \\
+ (D_i + \Delta D_i) \int_{t-\beta}^{t} x(s) ds + (B_i + \Delta B_i) u(t),
\]

where \(a(t), \beta, \gamma\) are time-varying discrete delay, constant distributed delay, and constant neutral delay, respectively. They may be unknown but they satisfy \(0 \leq a(t) \leq a_M, \beta(t) < d < 1, 0 \leq \beta \leq \beta_M, 0 < \gamma < \gamma_M\) where \(a_M, d, \beta_M, \gamma_M\) are known numbers. \(x(t) \in \mathbb{R}^n\) is the state and \(u(t) \in \mathbb{R}^m\) is the input. The matrices \(A_i, A_{di}, A_{mi}, B_i\) and \(D_i\) are of appropriate dimensions. \(r\) is the number of IF-THEN rule. \(M_{ij}\) is a fuzzy set and \(\xi_1, \ldots, \xi_p\) are premise variables. We set \(\xi = [\xi_1, \ldots, \xi_p]^T\) and \(\xi(t)\) is assumed to be available. The uncertain matrices are of the form

\[
\begin{bmatrix}
\Delta A_i(t) & \Delta A_{di}(t) & \Delta A_{mi}(t) & \Delta B_i(t) & \Delta D_i(t)
\end{bmatrix}
= H_i F_i(t) \begin{bmatrix} E_{i1} & E_{i2} & E_{i3} & E_{bi} & E_{di} \end{bmatrix}, \quad i = 1, \ldots, r
\]

where \(H_i, E_{i1}, E_{i2}, E_{i3}, E_{bi}\) and \(E_{di}\) are known matrices of appropriate dimensions, and each \(F_i(t)\) is unknown real time varying matrices satisfying

\[
F_i^T(t) F_i(t) \leq I, \quad i = 1, \ldots, r
\]

The system is defined as follows:

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t))(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-\alpha(t)) \\
&\quad + (D_i + \Delta D_i) \int_{t-\beta}^{t} x(s) ds + (B_i + \Delta B_i) u(t), \\
y(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t)) C_i x(t)
\end{align*}
\]

where \(\lambda_i(\xi) = \frac{\mu_i(\xi)}{\sum_{j=1}^{p} \mu_j(\xi)}\), \(\mu_i(\xi) = \prod_{j=1}^{p} M_{ij}(\xi_j)\) and \(M_{ij}(\cdot)\) is the grade of the membership function of \(M_{ij}\). We assume \(\mu_i(\xi(t)) \geq 0, i = 1, \ldots, r, \sum_{j=1}^{p} \mu_j(\xi(t)) > 0\) for any \(\xi(t)\). Hence \(\lambda_i(\xi(t))\) satisfy \(\lambda_i(\xi(t)) \geq 0, i = 1, \ldots, r, \sum_{j=1}^{p} \lambda_j(\xi(t)) = 1\) for any \(\xi(t)\). We consider the sampled-data control input. It may be represented as delayed control as follows:

\[
u(t) = u_d(t_k) = u_d(t - (t - t_k)) = u_d(t - h(t)), \quad t_k \leq t \leq t_{k+1}
\]

where \(u_d\) is a zero-order control signal and the time-varying delay \(0 \leq h(t) = t - t_k\) is piecewise linear with the derivative \(h(t) = 1\) for \(t \neq t_k\). A sampling time \(t_k\) is the time-varying sampling instant satisfying \(0 < t_1 < t_2 < \ldots < t_k < \ldots\). Sampling interval \(h_k = t_{k+1} - t_k\) may vary but it is bounded. Thus, we assume \(h(t) \leq t_{k+1} - t_k = h_k \leq h_M\) for all \(t_k\) where \(h_M\) is known constant. We consider the following rules for a controller:

\[
\text{IF } \xi_i(t_k) = M_{i1} \text{ and } \ldots \text{ and } \xi_p(t_k) = M_{ip}, \\
\text{THEN } u(t) = K_i x(t_k), \quad i = 1, \ldots, r
\]
where $K_i$ is to be determined. Then, the natural choice of a controller is given by

$$ u(t) = \sum_{i=1}^{r} \lambda_i(t_k) K_i x(t_k). $$

(2)

We represent a piecewise control law as a continuous-time one with a time-varying piecewise continuous (continuous from the right) delay $h(t)$. Hence, we look for a state feedback controller of the form

$$ u(t) = \sum_{i=1}^{r} \lambda_i(t_k) K_i x(t - h(t)). $$

(3)

that robustly stabilizes the system (1). The system is said to be robustly stable if it is asymptotically stable for all admissible uncertainties. The closed-loop system (1) with (3) becomes

$$ \dot{x}(t) - \sum_{i=1}^{r} \lambda_i(t_k)(A_{wi} + \Delta A_{wi})x(t - \gamma) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(t_k) \lambda_j(t_k) \{(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \alpha(t)) + (D_i + \Delta D_i) \int_{t-\beta}^{t} x(s)ds + (B_i + \Delta B_i)K_i x(t - h(t))\}. $$

When we consider a nominal system, we have

$$ \dot{x}(t) - \sum_{i=1}^{r} \lambda_i(t_k) A_{wi} x(t - \gamma) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(t_k) \lambda_j(t_k) \{(A_i)x(t) + A_{di}x(t - \alpha(t)) + D_i \int_{t-\beta}^{t} x(s)ds + B_i K_i x(t - h(t))\}. $$

(4)

3. Stability analysis

First, we make stability analysis of the nominal closed-loop system (4).

**Theorem 3.1** Given control gain matrices $K_i$, $i = 1, \ldots, r$, the closed-loop system (4) is asymptotically stable if there exist matrices $P_i > 0$, $R_i > 0$, $X > 0$, $Y_i > 0$, $i = 1, 2, 3$, $Q_i \geq 0$, $Z_i > 0$, $i = 1, 2, 3$, and

$$ N_{ij} = \begin{bmatrix} N^T_{1ij} & N^T_{2ij} & N^T_{3ij} & N^T_{4ij} & N^T_{5ij} & N^T_{6ij} & N^T_{7ij} & N^T_{8ij} & N^T_{9ij} \end{bmatrix}^T, $$

$$ S_{ij} = \begin{bmatrix} S^T_{1ij} & S^T_{2ij} & S^T_{3ij} & S^T_{4ij} & S^T_{5ij} & S^T_{6ij} & S^T_{7ij} & S^T_{8ij} & S^T_{9ij} \end{bmatrix}^T, $$

$$ M_{ij} = \begin{bmatrix} M^T_{1ij} & M^T_{2ij} & M^T_{3ij} & M^T_{4ij} & M^T_{5ij} & M^T_{6ij} & M^T_{7ij} & M^T_{8ij} & M^T_{9ij} \end{bmatrix}^T, $$

$$ V_{ij} = \begin{bmatrix} V^T_{1ij} & V^T_{2ij} & V^T_{3ij} & V^T_{4ij} & V^T_{5ij} & V^T_{6ij} & V^T_{7ij} & V^T_{8ij} & V^T_{9ij} \end{bmatrix}^T, $$

$$ W_{ij} = \begin{bmatrix} W^T_{1ij} & W^T_{2ij} & W^T_{3ij} & W^T_{4ij} & W^T_{5ij} & W^T_{6ij} & W^T_{7ij} & W^T_{8ij} & W^T_{9ij} \end{bmatrix}^T, $$

$$ O_{ij} = \begin{bmatrix} O^T_{1ij} & O^T_{2ij} & O^T_{3ij} & O^T_{4ij} & O^T_{5ij} & O^T_{6ij} & O^T_{7ij} & O^T_{8ij} & O^T_{9ij} \end{bmatrix}^T, $$

$$ T = \begin{bmatrix} T^T_1 & T^T_2 & T^T_3 & T^T_4 & T^T_5 & T^T_6 & T^T_7 & T^T_8 & T^T_9 \end{bmatrix}^T. $$

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such that

$$\Phi_{ij} = \begin{bmatrix} \Phi_{11ij} & \Phi_{12ij} \\ \Phi_{21ij} & \Phi_{22ij} \end{bmatrix} \leq 0, \ i, j = 1, \cdots, r$$

(5)

where

$$\Phi_{11ij} = \Phi_1 + \Phi_{21ij} + \Phi_{21ij}^T + \Phi_{31ij} + \Phi_{31ij}^T,$$

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{T12} & \Phi_{22} \\ 0 & 0 & -R & 0 \\ 0 & 0 & - (1 - d) Q_1 & 0 \\ 0 & 0 & 0 & 0 \\ \Phi_{12} & \Phi_{T12} & \Phi_{22} & \Phi_{22} \end{bmatrix},$$

$$\Phi_{11} = Q_1 + R + \beta M U + \beta \tilde{S} X,$$

$$\Phi_{199} = Q_2 + \alpha M Y_1 + \beta M Y_2 + \gamma M Y_3 + h M \left( Z_1 + Z_2 \right),$$

$$\Phi_{2ij} = \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} + O_{ij} + V_{ij} & - N_{ij} + S_{ij} & - M_{ij} - S_{ij} & - W_{ij} \\ - O_{ij} & 0 & - V_{ij} & 0 \end{bmatrix},$$

$$\Phi_{3ij} = \begin{bmatrix} - T A_i & - T B_i K_j & 0 & - T A_i \end{bmatrix},$$

$$\Phi_{12ij} = \begin{bmatrix} h_M N_{ij} & h_M S_{ij} & h_M M_{ij} & \alpha M W_{ij} & \beta M O_{ij} & \gamma M V_{ij} \end{bmatrix},$$

$$\Phi_{22} = \text{diag} \left[ - h_M Z_1, - h_M Z_1, - h_M Z_2, - \alpha M Y_1, - \beta M Y_2, - \gamma M Y_3 \right].$$

Proof: First, it follows from the Leibniz-Newton formula that the following equations hold for any matrices $N_{ij}, S_{ij}, M_{ij}, V_{ij}, W_{ij}$ and $O_{ij}$, the forms of which are given in Theorem 31.

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i (\xi(t)) \lambda_j (\xi(t)) \xi^T(t) N_{ij} \left[ x(t) - x(t - h(t)) - \int_{t-h(t)}^{t} \dot{x}(s) ds \right] = 0,$$

(6)

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i (\xi(t)) \lambda_j (\xi(t)) \xi^T(t) S_{ij} \left[ x(t - h(t)) - x(t - h_M) - \int_{t-h_M}^{t-h(t)} \dot{x}(s) ds \right] = 0,$$

(7)

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i (\xi(t)) \lambda_j (\xi(t)) \xi^T(t) M_{ij} \left[ x(t) - x(t - M) - \int_{t-M}^{t} \dot{x}(s) ds \right] = 0,$$

(8)

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i (\xi(t)) \lambda_j (\xi(t)) \xi^T(t) V_{ij} \left[ x(t) - x(t - \gamma) - \int_{t-\gamma}^{t} \dot{x}(s) ds \right] = 0,$$

(9)

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i (\xi(t)) \lambda_j (\xi(t)) \xi^T(t) W_{ij} \left[ x(t) - x(t - \alpha(t)) - \int_{t-\alpha(t)}^{t} \dot{x}(s) ds \right] = 0,$$

(10)

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i (\xi(t)) \lambda_j (\xi(t)) \xi^T(t) O_{ij} \left[ x(t) - x(t - \beta) - \int_{t-\beta}^{t} \dot{x}(s) ds \right] = 0,$$

(11)
where
\[
\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_M) & x^T(t-a(t)) & x^T(t-\beta) \\
\end{bmatrix}^T.
\]

It is also clear from the closed-loop system (4) that the following is true for any matrix \( T \).

\[
2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\zeta(t)) \lambda_j(\zeta(t)) \zeta_i^T(t) T \left[ \dot{x}(t) - A_i x(t) \right] - A_{di} \dot{x}(t-a(t)) - A_{di} \dot{x}(t-\gamma) - D_i \int_{t-\beta}^{t} x(s) ds - B_i K_i \dot{x}(t-h(t)) = 0. \tag{12}
\]

Now, we consider the following Lyapunov-Krasovskii functional:

\[
V(x_t) = V_1(x) + V_2(x_t) + V_3(x_t) + V_4(x_t)
\]

where \( x_t = x(t+\theta) \), \(-\max(h_M, \alpha_M, \beta_M) \leq \theta \leq 0, \)

\[
V_1(x) = x^T(t) P_1 x(t) + \left[ \int_{t-\beta}^{t} x(s) ds \right]^T P_2 \left[ \int_{t-\beta}^{t} x(s) ds \right],
\]

\[
V_2(x_t) = \int_{t-a(t)}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\gamma}^{t} x^T(s) Q_2 \dot{x}(s) ds + \int_{t-h_M}^{t} x^T(s) R x(s) ds,
\]

\[
V_3(x_t) = \int_{t-\beta}^{t} \int_{t+\theta}^{t} x^T(s) U X(s) ds d\theta + \int_{-\alpha_M}^{0} \int_{t+\theta}^{t} x^T(s) Y_1 \dot{x}(s) ds d\theta
\]

\[
+ \int_{-\beta}^{0} \int_{t+\theta}^{t} x^T(s) Y_2 \dot{x}(s) ds d\theta + \int_{-\gamma}^{0} \int_{t+\theta}^{t} x^T(s) Y_3 \dot{x}(s) ds d\theta
\]

\[
+ \int_{-h_M}^{0} \int_{t+\theta}^{t} x^T(s) (Z_1 + Z_2) \dot{x}(s) ds d\theta,
\]

\[
V_4(x_t) = \int_{t-\beta}^{t} \left[ \int_{t+\theta}^{t} x^T(s) ds \right] X \left[ \int_{t+\theta}^{t} x(s) ds \right] d\theta + \int_{0}^{\beta} \int_{t}^{t+\theta} (s-t+\theta) x^T(s) X x(s) ds d\theta,
\]

and \( P_1 > 0, R \geq 0, U > 0, X > 0, Y_i > 0, i = 1,2,3, Q_i \geq 0, Z_i > 0, i = 1,2 \) are to be determined.

We take the derivative of \( V(x_t) \) with respect to \( t \) along the solution of the system (4) and add
the left-hand-sides of (6)-(12):

\[
\dot{V}(x_t) \leq 2x^T(t)P_1x(t) + 2x^T(t)P_2 \int_{t-\beta}^{t} x(s)ds - 2x^T(t - \beta)P_2 \int_{t-\beta}^{t} x(s)ds \\
+ \dot{x}^T(t)(Q_1 + R + \beta_MU + \beta_{XM}X)x(t) - (1 - d)x^T(t - \alpha(t))Q_1x(t - \alpha(t)) \\
- \dot{x}^T(t - \gamma)Q_2\dot{x}(t - \gamma) - x^T(t - h_M)Rx(t - h_M) \\
- \left[ \int_{t-\beta}^{t} x(s)ds \right]^T \frac{1}{\beta_M} \left[ \int_{t-\beta}^{t} x(s)ds \right] \\
+ \dot{x}^T(t)\left[ Q_2 + \alpha_MY_1 + \beta_MY_2 + \gamma_MY_3 + h_M(Z_1 + Z_2) \right]\dot{x}(t) \\
- \int_{t-h_M}^{t} x^T(s)Y_1x(s)ds - \int_{t-\beta}^{t-\gamma} \dot{x}^T(s)Y_2\dot{x}(s)ds - \int_{t-\gamma}^{t} \dot{x}^T(s)Y_3\dot{x}(s)ds \\
- \int_{t-h_M}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h_M}^{t-\gamma} \dot{x}^T(s)Z_1\dot{x}(s)ds \\
- \int_{t-h_M}^{t} \dot{x}^T(s)Z_2\dot{x}(s)ds - \left[ \int_{t-\beta}^{t} x(s)ds \right]X \left[ \int_{t-\beta}^{t} x(s)ds \right] \\
+ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_h))\xi^T(t)N_{ij} \left[ x(t) - x(t - h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds \right] \\
+ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_h))\xi^T(t)S_{ij} \left[ x(t - h(t)) - x(t - h_M) - \int_{t-h_M}^{t} \dot{x}(s)ds \right] \\
+ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_h))\xi^T(t)M_{ij} \left[ x(t) - x(t - h_M) - \int_{t-h_M}^{t} \dot{x}(s)ds \right] \\
+ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_h))\xi^T(t)V_{ij} \left[ x(t - h(t)) - x(t - \gamma) - \int_{t-\gamma}^{t} \dot{x}(s)ds \right] \\
+ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_h))\xi^T(t)W_{ij} \left[ x(t) - x(t - \alpha(t)) - \int_{t-\alpha(t)}^{t} \dot{x}(s)ds \right] \\
+ 2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t))\lambda_j(\xi(t_h))\xi^T(t)O_{ij} \left[ x(t) - x(t - \beta) - \int_{t-\beta}^{t} \dot{x}(s)ds \right] \\
- A_{\alpha(t)}\dot{x}(t - \gamma) - D_1\int_{t-\beta}^{t} x(s)ds - B_1Kx(t - h(t)) \right] \\
\leq \int_{t-h(t)}^{t} \left[ \xi^T(t)N_{ij} + \dot{x}^T(s)Z_1 \right] Z_1^{-1} \left[ N_{ij}T_\xi(t) + Z_1T_\xi(s) \right]ds \\
- \int_{t-h_M}^{t-\gamma} \left[ \xi^T(t)S_{ij} + \dot{x}^T(s)Z_1 \right] Z_1^{-1} \left[ S_{ij}T_\xi(t) + Z_1T_\xi(s) \right]ds \\
- \int_{t-h_M}^{t-\gamma} \left[ \xi^T(t)M_{ij} + \dot{x}^T(s)Z_2 \right] Z_2^{-1} \left[ M_{ij}T_\xi(t) + Z_2T_\xi(s) \right]ds \\
- \int_{t-\gamma}^{t} \left[ \xi^T(t)V_{ij} + \dot{x}^T(s)Y_1 \right] Y_1^{-1} \left[ V_{ij}T_\xi(t) + Y_1T_\xi(s) \right]ds \\
- \int_{t-\beta}^{t} \left[ \xi^T(t)W_{ij} + \dot{x}^T(s)Y_2 \right] Y_2^{-1} \left[ W_{ij}T_\xi(t) + Y_2T_\xi(s) \right]ds 
\]
where

$$\Psi_{ij} = \Phi_{1ij} + h_M(N_i'Z_i - N_i') + S_i'Z_i - S_i' + M_i'Z_i - M_i') + \alpha MW_i'Y_i - W_i'Y_i' + \beta M'Y_i' + \gamma M'Y_i'Y_i'V_i'Y_i'V_i'. $$

Now, if (5) is satisfied, then by Schur complement formula we have

$$\Psi_{ij} < 0, \quad i, j = 1, \ldots, r.$$  

If (14) holds, we have

$$\sum_{i=1}^r \sum_{j=1}^r \lambda_i(\zeta(t)) \lambda_j(\zeta(t)) \zeta^T(t) \Psi_{ij} \zeta(t) < 0,$$

which implies that $\bar{V}(x_i) < 0$ because $Y_i > 0$, $Z_i > 0$, $i = 1, 2$, and the last five terms in (13) are all less than 0. This proves that conditions (5) suffice to show the asymptotic stability of the system (4).

4. Sampled-data control design

In this section, we seek a design method of a sampled-data control for fuzzy time-delay systems based on Theorem 3.1. Unfortunately, however, Theorem 3.1 does not give feasible LMI conditions for obtaining state feedback control gain matrices $K_i$. To this end, we adopt an appropriate congruence transformation to obtain feasible LMI conditions and a design method of a sampled-data state feedback controller.

**Theorem 4.1** Given scalars $p_i$, $i = 1, \ldots, 9$, the sampled-data controller (2) asymptotically stabilizes the nominal system (4) if there exist matrices $P_i > 0$, $R > 0$, $Q_i > 0$, $\bar{V}_i > 0$, $i = 1, 2, 3$, $Q_i > 0$, $Z_i > 0$, $i = 1, 2, L$, $G_{ij}, j = 1, \ldots, r$,

$$\bar{N}_{ij} = \begin{bmatrix} \bar{N}_{ij}^T & \bar{N}_{2ij} & \bar{N}_{3ij}^T & \bar{N}_{4ij} & \bar{N}_{5ij} & \bar{N}_{6ij} & \bar{N}_{7ij} & \bar{N}_{8ij} & \bar{N}_{9ij} \end{bmatrix}^T,$$

$$\bar{S}_{ij} = \begin{bmatrix} \bar{S}_{ij}^T & \bar{S}_{2ij} & \bar{S}_{3ij}^T & \bar{S}_{4ij} & \bar{S}_{5ij} & \bar{S}_{6ij} & \bar{S}_{7ij} & \bar{S}_{8ij} & \bar{S}_{9ij} \end{bmatrix}^T,$$

$$\bar{M}_{ij} = \begin{bmatrix} \bar{M}_{ij}^T & \bar{M}_{2ij} & \bar{M}_{3ij}^T & \bar{M}_{4ij} & \bar{M}_{5ij} & \bar{M}_{6ij} & \bar{M}_{7ij} & \bar{M}_{8ij} & \bar{M}_{9ij} \end{bmatrix}^T,$$

$$\bar{V}_{ij} = \begin{bmatrix} \bar{V}_{ij}^T & \bar{V}_{2ij} & \bar{V}_{3ij}^T & \bar{V}_{4ij} & \bar{V}_{5ij} & \bar{V}_{6ij} & \bar{V}_{7ij} & \bar{V}_{8ij} & \bar{V}_{9ij} \end{bmatrix}^T,$$

$$\bar{W}_{ij} = \begin{bmatrix} \bar{W}_{ij}^T & \bar{W}_{2ij} & \bar{W}_{3ij}^T & \bar{W}_{4ij} & \bar{W}_{5ij} & \bar{W}_{6ij} & \bar{W}_{7ij} & \bar{W}_{8ij} & \bar{W}_{9ij} \end{bmatrix}^T,$$

$$\bar{O}_{ij} = \begin{bmatrix} \bar{O}_{1ij}^T & \bar{O}_{2ij} & \bar{O}_{3ij}^T & \bar{O}_{4ij} & \bar{O}_{5ij} & \bar{O}_{6ij} & \bar{O}_{7ij} & \bar{O}_{8ij} & \bar{O}_{9ij} \end{bmatrix}^T,$$

such that

$$\Theta_{ij} = \begin{bmatrix} \Theta_{11ij} & \Theta_{12ij} \\ \Theta_{12ij} & \Theta_{22} \end{bmatrix} < 0, \quad i, j = 1, \ldots, r$$  

(15)
where

\[
\begin{align*}
\Theta_{11ij} &= \Theta_1 + \Theta_{2ij} + \Theta_{2ij}^T + \Theta_{3ij} + \Theta_{3ij}^T, \\
\Theta_1 &= \begin{bmatrix}
\Theta_{111} & 0 & 0 & 0 & 0 & 0 & \tilde{P}_2 & \tilde{P}_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\tilde{R} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -(1-d)\tilde{Q}_1 & 0 & 0 & 0 & -\tilde{P}_2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\tilde{P}_2 & 0 & 0 & 0 & 0 & -\tilde{P}_2 & 0 & 0 \\
\tilde{P}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \\
\Theta_{11} &= Q_1 + \tilde{R} + \beta_M\tilde{Y}_1 + \beta_M\tilde{Y}_2 + \gamma_M\tilde{Y}_3 + h_M(Z_1 + Z_2), \\
\Theta_{199} &= Q_2 + \alpha_M\tilde{Y}_1 + \beta_M\tilde{Y}_2 + \gamma_M\tilde{Y}_3 + h_M(Z_1 + Z_2), \\
\Theta_{2ij} &= \begin{bmatrix}
\tilde{N}_{ij} + \tilde{M}_{ij} + \tilde{W}_{ij} + \tilde{O}_{ij} + \tilde{V}_{ij} - \tilde{N}_{ij} - \tilde{S}_{ij} - \tilde{M}_{ij} - \tilde{S}_{ij} - \tilde{W}_{ij} \\
-\tilde{O}_{ij} & 0 & -\tilde{V}_{ij} & 0 & 0
\end{bmatrix}, \\
\Theta_{3ij} &= \begin{bmatrix}
\rho_1 A_i L_i^T & \rho_1 B_i G_i & 0 & \rho_1 A_i L_i^T & 0 & \rho_1 D_i L_i^T & -\rho_1 L_i^T \\
\rho_2 A_i L_i^T & \rho_2 B_i G_i & 0 & \rho_2 A_i L_i^T & 0 & \rho_2 D_i L_i^T & -\rho_2 L_i^T \\
\rho_3 A_i L_i^T & \rho_3 B_i G_i & 0 & \rho_3 A_i L_i^T & 0 & \rho_3 D_i L_i^T & -\rho_3 L_i^T \\
\rho_4 A_i L_i^T & \rho_4 B_i G_i & 0 & \rho_4 A_i L_i^T & 0 & \rho_4 D_i L_i^T & -\rho_4 L_i^T \\
\rho_5 A_i L_i^T & \rho_5 B_i G_i & 0 & \rho_5 A_i L_i^T & 0 & \rho_5 D_i L_i^T & -\rho_5 L_i^T \\
\rho_6 A_i L_i^T & \rho_6 B_i G_i & 0 & \rho_6 A_i L_i^T & 0 & \rho_6 D_i L_i^T & -\rho_6 L_i^T \\
\rho_7 A_i L_i^T & \rho_7 B_i G_i & 0 & \rho_7 A_i L_i^T & 0 & \rho_7 D_i L_i^T & -\rho_7 L_i^T \\
\rho_8 A_i L_i^T & \rho_8 B_i G_i & 0 & \rho_8 A_i L_i^T & 0 & \rho_8 D_i L_i^T & -\rho_8 L_i^T
\end{bmatrix}, \\
\Theta_{12ij} &= \begin{bmatrix}
h_M\tilde{N}_{ij} & h_M\tilde{S}_{ij} & h_M\tilde{M}_{ij} & \alpha_M\tilde{W}_{ij} & \beta_M\tilde{O}_{ij} & \gamma_M\tilde{V}_{ij}
\end{bmatrix}, \\
\Theta_{22} &= \text{diag}\begin{bmatrix}
h_MZ_1 & -h_MZ_1 & -h_MZ_2 & -\alpha_M\tilde{Y}_1 & -\beta_M\tilde{Y}_2 & -\gamma_M\tilde{Y}_3
\end{bmatrix}.
\end{align*}
\]

In this case, state feedback control gains in (2) are given by

\[
K_i = G_i L_i^{-T}, \quad i = 1, \ldots, r.
\] (16)

**Proof:** We let \(T_i = \rho_i \tilde{L}_i, \quad i = 1, \ldots, r\), the feedback law is defined later, and substitute them into (5). If (5) holds, it follows that (9,9)-block of \(\Phi_{11ij}\) must be negative definite. It follows that \(T_9 + T_9^T = \rho_9 (\tilde{L}_9 + \tilde{L}_9^T) < 0\), which implies that \(\tilde{L}_9\) is nonsingular. Then, we define \(L = L^{-1}\) and calculate \(\Theta_{ij} = \Sigma \Phi_{ij} \Sigma^T\) with \(\Sigma = \text{diag}\left[ L L L L L L L L L L L L L L L L\right].\) Defining \(P_i = LP_i L_i^T\), \(\tilde{R} = LRL_i^T\), \(\tilde{X} = LXL_i^T\), \(\tilde{Y}_i = LY_i L_i^T\), \(\tilde{Z}_i = LZ_i L_i^T\), \(\tilde{Q}_i = LQ_i L_i^T\), \(\tilde{Z}_i = LZ_i L_i^T\), \(\tilde{N}_{ij} = LN_{ij}\tilde{L}_i, \tilde{S}_{ij} = LS_{ij}\tilde{L}_i, \tilde{M}_{ij} = LM_{ij}\tilde{L}_i, \tilde{W}_{ij} = LW_{ij}\tilde{L}_i, \tilde{O}_{ij} = LO_{ij}\tilde{L}_i, \tilde{V}_{ij} = LV_{ij}\tilde{L}_i, \tilde{O}_{ij} = LO_{ij}\tilde{L}_i\), and \(i, j = 1, \ldots, r, k = 1, \ldots, 9\), we obtain \(\Theta_{ij}\) in (15) where we let \(G_i = K_i L_i^T.\) If the conditions (15) hold, state feedback control gain matrix \(K_i\) is obviously given by (16).

We extend the result to the case of the uncertain system (1). The following lemma is necessary to prove Theorem 4.3.
Lemma 4.2 \(((10))\) Given matrices $Q = Q^T$, $H$, $E$ and $R = R^T > 0$ of appropriate dimensions

$$Q + HF(t)E + E^TF^T(t)H^T < 0$$

for all $F(t)$ satisfying $F^T(t)F(t) \leq R$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$Q + \frac{1}{\varepsilon}HH^T + \varepsilon E^TRE < 0.$$

Theorem 4.3 Given scalars $\rho_{ij}$, $i = 1, \cdots, 9$, the sampled-data controller (2) robustly stabilizes the uncertain system (1) if there exist matrices $\bar{P}_i > 0$, $\bar{Q}_i \geq 0$, $\bar{Z}_i > 0$, $i = 1, 2, \bar{R} \geq 0$, $\bar{X} > 0$, $\bar{Y}_i > 0$, $i = 1, 2, 3$, $L_i$, $G_{ij}$, $j = 1, \cdots, r$, and

\[
\bar{N}_{ij} = \begin{bmatrix} \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T & \bar{N}_{ij}^T \end{bmatrix}^T,
\]
\[
\bar{S}_{ij} = \begin{bmatrix} \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T & \bar{S}_{ij}^T \end{bmatrix}^T,
\]
\[
\bar{M}_{ij} = \begin{bmatrix} \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T & \bar{M}_{ij}^T \end{bmatrix}^T,
\]
\[
\bar{V}_{ij} = \begin{bmatrix} \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T & \bar{V}_{ij}^T \end{bmatrix}^T,
\]
\[
\bar{W}_{ij} = \begin{bmatrix} \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T & \bar{W}_{ij}^T \end{bmatrix}^T,
\]
\[
\bar{O}_{ij} = \begin{bmatrix} \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T & \bar{O}_{ij}^T \end{bmatrix}^T,
\]

and scalars $\varepsilon_{ij} > 0$, $i, j = 1, \cdots, r$ such that

\[
\begin{bmatrix} \Theta_{ij} + \varepsilon_{ij} \bar{R}_i \bar{F}_i - \bar{E}_{ij}^T & \bar{E}_{ij}^T \\ \bar{E}_{ij} & -\varepsilon_{ij} I \end{bmatrix} < 0, \ i, j = 1, \cdots, r \tag{17}
\]

where $\Theta_{ij}$ is given in Theorem 4.1, and

\[
\bar{R}_i = -\begin{bmatrix} \rho_1 H_i^T & \rho_2 H_i^T & \rho_3 H_i^T & \rho_4 H_i^T & \rho_5 H_i^T & \rho_6 H_i^T & \rho_7 H_i^T & \rho_8 H_i^T \end{bmatrix}^T,
\]
\[
\bar{E}_{ij} = \begin{bmatrix} E_{1i}L_i^T & E_{1i}G_{ij} & 0 & E_{2i}L_i^T & 0 & E_{3i}L_i^T & 0 & E_{4i}L_i^T & 0 & E_{5i}L_i^T \end{bmatrix}^T.
\]

In this case, state feedback control gains in (2) are given by (16).

**Proof:** Replacing $A_i$, $A_{di}$, $A_{mi}$, $B_i$, $D_i$ by $A_i + H_i F_i(t) E_{1i}$, $A_{di} + H_i F_i(t) E_{2i}$, $A_{mi} + H_i F_i(t) E_{3i}$, $B_i + H_i F_i(t) E_{4i}$, $D_i + H_i F_i(t) E_{5i}$, we have

\[
\Theta_{ij} + \bar{H}_i F_i(t) \bar{E}_{ij} + (\bar{H}_i F_i(t) \bar{E}_{ij})^T < 0, \ i, j = 1, \cdots, r.
\]

It follows from Lemma 4.2 that the above LMIs hold if and only if there exist $\varepsilon_{ij} > 0$ such that

\[
\Theta_{ij} + \varepsilon_{ij} \bar{H}_i \bar{H}_i + \frac{1}{\varepsilon_{ij}} \bar{E}_{ij} \bar{E}_{ij} < 0, \ i, j = 1, \cdots, r.
\]

Applying the Schur complement formula, we have (17).
5. Numerical examples for controller design

Let us design robust sampled-data controllers for the system (1) with the following matrices.

\[
A_1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1.5 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -1 & -1 \\ 0 & -0.9 \end{bmatrix}, \quad A_{i2} = \begin{bmatrix} -1 & -1 \\ 0 & -1.4 \end{bmatrix},
\]

\[
A_{n1} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad A_{n2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 1.2 \end{bmatrix},
\]

\[
D_1 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}, \quad H_1 = H_2 = 0.2I, \quad E_{11} = E_{12} = 0.2I,
\]

\[
E_{21} = E_{22} = 0.2I, \quad E_{31} = E_{32} = 0.1I, \quad E_{i1} = E_{i2} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad E_{di} = E_{d2} = 0.1I.
\]

The grades are given by \( \lambda_1(x_1) = \sin^2(x_1) \) and \( \lambda_2(x_1) = 1 - \lambda_1(x_1) \). The maximum upper bound of the sampling time \( h_M = 0.1 \) and \( d = 0.5 \) are assumed. First, we let \( \beta_M = \gamma_M = 0.1 \). Theorem 4.3 with \( p_1 = 5.46, \quad p_2 = -0.01, \quad p_3 = -2.19, \quad p_4 = 0.60, \quad p_5 = -0.01, \quad p_6 = -0.01, \quad p_7 = 0.50, \quad p_8 = 0.10, \quad p_9 = 1.96 \) guarantees the existence of the sampled-data controller for the maximum upper bound of the time-delay \( \alpha_M = 0.42 \). In this case, control gains in (2) are given by

\[
K_1 = \begin{bmatrix} -0.1800 \\ -0.9934 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.1808 \\ -0.9942 \end{bmatrix}.
\]

Next, we let \( \alpha_M = \gamma_M = 0.1 \). Theorem 4.3 with \( p_1 = 5.74, \quad p_2 = 0.50, \quad p_3 = -2.19, \quad p_4 = -0.60, \quad p_5 = -0.01, \quad p_6 = -0.42, \quad p_7 = -0.50, \quad p_8 = 0.16, \quad p_9 = 1.96 \) gives a robust sampled-data controller for the maximum upper bound \( \beta_M = 3.43 \). In this case, control gains in (2) are given by

\[
K_1 = \begin{bmatrix} 0.1794 \\ -2.6198 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.1795 \\ -2.6194 \end{bmatrix}.
\]

Finally, we let \( \alpha_M = \beta_M = 0.1 \). Theorem 4.3 with \( p_1 = 4.74, \quad p_2 = -0.01, \quad p_3 = -2.19, \quad p_4 = -0.60, \quad p_5 = -0.01, \quad p_6 = 0.01, \quad p_7 = -0.50, \quad p_8 = 0.07, \quad p_9 = 1.96 \) gives a robust sampled-data controller for the maximum upper bound \( \gamma_M = 2.90 \). In this case, control gains in (2) are given by

\[
K_1 = \begin{bmatrix} -0.0265 \\ -0.7535 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.0260 \\ -0.7515 \end{bmatrix}.
\]

6. Application to observer design

In this section, using the results in the previous sections we consider an observer design for the system (1), which estimates the state variables of the system using sampled-data measurement outputs. Here, we assume that the system does not contain any uncertain parameters so that it is given by

\[
\dot{x}(t) - \sum_{i=1}^{r} \lambda_i(x(t))A_m \dot{x}(t - \gamma) = \sum_{i=1}^{r} \lambda_i(x(t)) \left\{ A_i x(t) + A_{d1} \dot{x}(t - \alpha(t)) + D_1 \int_{t-\beta}^{t} x(s) ds + B_i u(t) \right\}, \quad (18)
\]

\[
y(t) = \sum_{i=1}^{r} \lambda_i(x(t))C_i x(t) \quad (19)
\]

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where all the time delays are assumed to be measurable. The sampled-data measurement output may be represented as delayed measurement as follows:

\[ y(t) = y_d(t) = y_d(t - (t - t_k)) = y_d(t - h(t)), \quad t_k \leq t \leq t_{k+1} \]

where \( y_d \) is a zero-order measurement signal and the time-varying delay \( 0 \leq h(t) = t - t_k \) is piecewise linear with the derivative \( h(t) = 1 \) for \( t \neq t_k \) as before. We consider the following rules for a system to estimate the state variables:

**IF** \( \xi(t_k) \) is \( M_{i1} \) and \( \ldots \) and \( \xi_p(t_k) \) is \( M_{ip} \),

**THEN** \[
\dot{\hat{x}}(t) - A_{ni} \dot{\hat{x}}(t - \gamma) = A_{ni} \dot{\hat{x}}(t) + A_{di} \dot{\hat{x}}(t - \alpha(t)) + D_i \int_{t-\beta}^{t} \dot{\hat{x}}(s) ds + B_i u(t) + R_i (y(t_k) - C_i \dot{\hat{x}}(t_k)), \quad i = 1, \ldots, r
\]

where \( \hat{x} \) is the estimated state and \( R = \sum_{i=1}^{r} \lambda_i(\xi(t_k)) R_i \) is an observer gain to be determined. Then, the overall system is given by

\[
\dot{\hat{x}}(t) - \sum_{i=1}^{r} \lambda_i(\xi(t)) A_{ni} \dot{\hat{x}}(t - \gamma) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t)) \lambda_j(\xi(t_k)) \{ A_{i} \dot{\hat{x}}(t) + A_{di} \dot{\hat{x}}(t - \alpha(t)) + D_i \int_{t-\beta}^{t} \dot{\hat{x}}(s) ds + B_i u(t) + R_j (y(t_k) - C_i \dot{\hat{x}}(t_k)) \}.
\]

(20)

where we see the measurement output as

\[ y(t) = \sum_{i=1}^{r} \lambda_i(\xi(t_k)) C_i x(t - h(t)). \]

It follows from (18), (19) and (20) that the error \( e(t) = x(t) - \hat{x}(t) \) satisfies

\[
\dot{e}(t) - \sum_{i=1}^{r} \lambda_i(\xi(t)) A_{ni} e(t - \gamma) = \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t)) \lambda_j(\xi(t_k)) \{ A_{i} e(t) + A_{di} e(t - \alpha(t)) + D_i \int_{t-\beta}^{t} e(s) ds - \bar{R}_i C_i e(t_k) \}.
\]

(21)

We shall find conditions for (21) to be asymptotically stable. In this case, (20) becomes an observer for the system (18) and (19).

The following theorem gives conditions for the error system (21) to be asymptotically stable.
Theorem 6.1 Given control gain matrices \( K_i, \) \( i = 1, \ldots, r \), the error system (21) is asymptotically stable if there exist matrices \( P_1 > 0, \) \( R > 0, \) \( X > 0, \) \( Y_i > 0, \) \( i = 1, 2, 3, \) \( Q_j > 0, \) \( Z_i > 0, \) \( i = 1, 2, \) and

\[
N_{ij} = \begin{bmatrix} N_{11j}^T & N_{21j}^T & N_{31j}^T & N_{41j}^T & N_{51j}^T & N_{61j}^T & N_{71j}^T & N_{81j}^T & N_{91j}^T \end{bmatrix}^T, \\
S_{ij} = \begin{bmatrix} S_{11j}^T & S_{21j}^T & S_{31j}^T & S_{41j}^T & S_{51j}^T & S_{61j}^T & S_{71j}^T & S_{81j}^T & S_{91j}^T \end{bmatrix}^T, \\
M_{ij} = \begin{bmatrix} M_{11j}^T & M_{21j}^T & M_{31j}^T & M_{41j}^T & M_{51j}^T & M_{61j}^T & M_{71j}^T & M_{81j}^T & M_{91j}^T \end{bmatrix}^T, \\
V_{ij} = \begin{bmatrix} V_{11j}^T & V_{21j}^T & V_{31j}^T & V_{41j}^T & V_{51j}^T & V_{61j}^T & V_{71j}^T & V_{81j}^T & V_{91j}^T \end{bmatrix}^T, \\
W_{ij} = \begin{bmatrix} W_{11j}^T & W_{21j}^T & W_{31j}^T & W_{41j}^T & W_{51j}^T & W_{61j}^T & W_{71j}^T & W_{81j}^T & W_{91j}^T \end{bmatrix}^T, \\
O_{ij} = \begin{bmatrix} O_{11j}^T & O_{21j}^T & O_{31j}^T & O_{41j}^T & O_{51j}^T & O_{61j}^T & O_{71j}^T & O_{81j}^T & O_{91j}^T \end{bmatrix}^T, \quad i, j = 1, \ldots, r,
\]

such that

\[
T = \begin{bmatrix} T_1^T & T_2^T & T_3^T & T_4^T & T_5^T & T_6^T & T_7^T & T_8^T \end{bmatrix}^T,
\]

where

\[
\Phi_{11i} = \Phi_1 + \Phi_{21j} + \Phi_{31j} + \Phi_{41j},
\]

\[
\Phi_1 = \begin{bmatrix} \Phi_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_2 & 0 & P_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
\]

\[
\Phi_{111} = Q_1 + R + \beta_M U + \beta_M U X,
\]

\[
\Phi_{199} = Q_2 + \alpha_M Y_1 + \beta_M Y_2 + \gamma M Y_3 + h_M (Z_1 + Z_2),
\]

\[
\Phi_{21j} = \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} + O_{ij} + V_{ij} & -N_{ij} + S_{ij} & -M_{ij} - S_{ij} & -W_{ij} \\ -O_{ij} & 0 & -V_{ij} & 0 & 0 \end{bmatrix},
\]

\[
\Phi_{31j} = \begin{bmatrix} -T A_i & T K_i C_i & 0 & -T A_{ii} & 0 & -T A_{ni} & 0 & -T D_i & T \end{bmatrix},
\]

\[
\Phi_{12j} = \begin{bmatrix} h_M N_{ij} & h_M S_{ij} & h_M M_{ij} & \alpha_M W_{ij} & \beta_M O_{ij} & \gamma M V_{ij} \end{bmatrix},
\]

\[
\Phi_{22} = \text{diag}\left[ -h_M Z_1 & -h_M Z_1 & -h_M Z_2 & -\alpha_M Y_1 & -\beta_M Y_2 & -\gamma M Y_3 \right].
\]
Theorem 6.1 still does not propose an observer design method. Hence, we give the following

**Proof:** Proof is similar to that of Theorem 3.1. We first note that the following is true for any matrix $T$.

$$2 \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t)) \lambda_j(\xi(t)) \xi^T(t) T [\xi(t) - A_1 \xi(t)] - \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t)) \lambda_j(\xi(t)) \xi^T(t) \xi(t) T [\xi(t) - A_1 \xi(t)] = 0 \quad \text{(23)}$$

where

$$\xi(t) = \left[ e^T(t) \quad e^T(t-h(t)) \quad e^T(t-h_M) \quad e^T(t-a(t)) \quad e^T(t-\beta) \quad e^T(t-\gamma) \quad \int_{t-\beta}^{t} e^T(s) ds \quad e^T(t) \right]^T.$$

Now, we take the derivative of $V(\epsilon(t))$, which is defined as $V(x_1)$ with replacing $x_1$ by $e_t$, with respect to $t$ along the solution of the error system (21) and add the left-hand-sides of (6)-(11) with replacing $x$ by $e$ and (23):

$$V(\epsilon_t) \leq \sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t)) \lambda_j(\xi(t)) \left\{ \xi^T(t) \Xi_{ij} \xi(t) \right\}$$

$$= \int_{t-h(t)}^{t} \left[ \xi^T(t) N_{ij} + \xi^T(s) Z_1 \right] Z_1^{-1} \left[ N_{ij}^T \xi(t) + Z_1 \dot{\xi}(s) \right] ds$$

$$= \int_{t-h_M}^{t-\gamma} \left[ \xi^T(t) S_{ij} + \xi^T(s) Z_1 \right] Z_1^{-1} \left[ S_{ij}^T \xi(t) + Z_1 \dot{\xi}(s) \right] ds$$

$$= \int_{t-\beta}^{t-\gamma} \left[ \xi^T(t) M_{ij} + \xi^T(s) Z_2 \right] Z_2^{-1} \left[ M_{ij}^T \xi(t) + Z_2 \dot{\xi}(s) \right] ds$$

$$= \int_{t-a(t)}^{t-\gamma} \left[ \xi^T(t) W_{ij} + \xi^T(s) Y_1 \right] Y_1^{-1} \left[ W_{ij}^T \xi(t) + Y_1 \dot{\xi}(s) \right] ds$$

$$= \int_{t-\gamma}^{t} \left[ \xi^T(t) V_{ij} + \xi^T(s) Y_2 \right] Y_2^{-1} \left[ V_{ij}^T \xi(t) + Y_2 \dot{\xi}(s) \right] ds$$

where

$$\Xi_{ij} = \Theta_{bij} + h_M N_{ij}^T Z_1^{-1} S_{ij} + S_{ij}^T Z_1^{-1} S_{ij} + M_{ij}^T Y_1^{-1} W_{ij} + \gamma M_{ij} Y_1^{-1} Y_2$$

Now, if (22) is satisfied, then by Schur complement formula we have

$$\Theta_{ij} < 0, \ i, j = 1, \ldots, r.$$  \quad \text{(25)}$$

If (25) holds, we have $\sum_{i=1}^{r} \sum_{j=1}^{r} \lambda_i(\xi(t)) \lambda_j(\xi(t)) \xi^T(t) \Psi_{ii} \xi(t) \leq 0$, which implies that $V(\epsilon(t)) < 0$ because $Y_i > 0, Z_i > 0, i = 1, 2$ and the last five terms in (24) are all less than 0. This proves that conditions (22) suffice to show the asymptotic stability of the system (21).

Theorem 6.1 still does not propose an observer design method. Hence, we give the following result.

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Theorem 6.2 Given scalars $\rho_i$, $i = 1, \ldots, 9$, (20) becomes an observer for the nominal system (18) and (19) if there exist matrices $P_1 > 0$, $R > 0$, $U > 0$, $X > 0$, $\gamma_i > 0$, $i = 1, 2, 3$, $Q_i > 0$, $Z_i > 0$, $i = 1, 2, L$, $G_i$, $j = 1, \ldots, r$,

$$
N_{ij} = \begin{bmatrix} N_{1ij}^T & N_{2ij}^T & N_{3ij}^T & N_{4ij}^T & N_{5ij}^T & N_{6ij}^T & N_{7ij}^T & N_{8ij}^T & N_{9ij}^T \end{bmatrix}^T,
$$

$$
S_{ij} = \begin{bmatrix} S_{1ij}^T & S_{2ij}^T & S_{3ij}^T & S_{4ij}^T & S_{5ij}^T & S_{6ij}^T & S_{7ij}^T & S_{8ij}^T & S_{9ij}^T \end{bmatrix}^T,
$$

$$
M_{ij} = \begin{bmatrix} M_{1ij}^T & M_{2ij}^T & M_{3ij}^T & M_{4ij}^T & M_{5ij}^T & M_{6ij}^T & M_{7ij}^T & M_{8ij}^T & M_{9ij}^T \end{bmatrix}^T,
$$

$$
V_{ij} = \begin{bmatrix} V_{1ij}^T & V_{2ij}^T & V_{3ij}^T & V_{4ij}^T & V_{5ij}^T & V_{6ij}^T & V_{7ij}^T & V_{8ij}^T & V_{9ij}^T \end{bmatrix}^T,
$$

$$
W_{ij} = \begin{bmatrix} W_{1ij}^T & W_{2ij}^T & W_{3ij}^T & W_{4ij}^T & W_{5ij}^T & W_{6ij}^T & W_{7ij}^T & W_{8ij}^T & W_{9ij}^T \end{bmatrix}^T,
$$

$$
O_{ij} = \begin{bmatrix} O_{1ij}^T & O_{2ij}^T & O_{3ij}^T & O_{4ij}^T & O_{5ij}^T & O_{6ij}^T & O_{7ij}^T & O_{8ij}^T & O_{9ij}^T \end{bmatrix}^T, i, j = 1, \ldots, r
$$

such that

$$
\Theta_{ij} = \begin{bmatrix} \Theta_{11ij} & \Theta_{12ij} \\ \Theta_{12ij} & \Theta_{22} \end{bmatrix} < 0, i, j = 1, \ldots, r
$$

(26)

where

$$
\Theta_{11ij} = \Theta_1 + \Theta_{2ij} + \Theta_{3ij} + \Theta_{4ij} + \Theta_{5ij}^T,
$$

$$
\Theta_1 = \begin{bmatrix} \Theta_{111} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -R & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1 - d)Q_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -P_2 \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_2 & 0 & 0 & 0 & 0 & -P_2 & 0 & 0 & -X - \frac{1}{b_3} U \\ P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
\Theta_{111} = Q_1 + R + \beta M U + \beta_3 X,
$$

$$
\Theta_{199} = Q_2 + \delta M Y_1 + \beta M Y_2 + \gamma M Y_3 + h M (Z_1 + Z_2),
$$

$$
\Theta_{2ij} = \begin{bmatrix} N_{ij} + M_{ij} + W_{ij} + O_{ij} + V_{ij} & -N_{ij} + S_{ij} & -M_{ij} - S_{ij} & -W_{ij} & -O_{ij} & 0 & -V_{ij} & 0 & 0 \end{bmatrix},
$$

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Robust Sampled-Data Control Design of Uncertain Fuzzy Systems with Discrete and Distributed Delays

In this paper, robust sampled-data control and observer design for uncertain fuzzy systems with discrete, neutral and distributed delays has been considered. Less conservative robust stability conditions were obtained as LMI conditions via time-varying delay system approach. Then, a controller design method was proposed via LMI conditions. As a dual result, an observer design method was also given. Finally, some examples were given to illustrate our design approach.

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9. References


Ferroelectric materials exhibit a wide spectrum of functional properties, including switchable polarization, piezoelectricity, high non-linear optical activity, pyroelectricity, and non-linear dielectric behaviour. These properties are crucial for application in electronic devices such as sensors, microactuators, infrared detectors, microwave phase filters and, non-volatile memories. This unique combination of properties of ferroelectric materials has attracted researchers and engineers for a long time. This book reviews a wide range of diverse topics related to the phenomenon of ferroelectricity (in the bulk as well as thin film form) and provides a forum for scientists, engineers, and students working in this field. The present book containing 24 chapters is a result of contributions of experts from international scientific community working in different aspects of ferroelectricity related to experimental and theoretical work aimed at the understanding of ferroelectricity and their utilization in devices. It provides an up-to-date insightful coverage to the recent advances in the synthesis, characterization, functional properties and potential device applications in specialized areas.

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